how to use DUAL BASE LOG LOG SLIDE RULES

by Professor Maurice L. Hartung*
The University of Chicago

Pickett
The World's Most Accurate
Slide Rules

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Preface

Your DUAL-BASE LOG LOG Slide Rule is a powerful modern design that has many features not found on traditional rules. These features make it easier to learn how to use the rule. They make it easier to remember the methods of solving problems as they arise in your work. They give you greater *range* on some scales, and greater *accuracy* in the solution of many problems.

If you are familiar with traditional slide rules, don't let the appearance of the DUAL-BASE rule "scare" you. You will quickly recognize the basic features of all slide rules as they have been improved in the design of this one. When you have become familiar with your DUAL-BASE rule, you will never again be satisfied to use a traditional rule. The problems of today require modern tools—better, faster, more accurate. Your DUAL-BASE rule is a *modern* tool.

A computer who must make many difficult calculations usually has a book of tables of the elementary mathematical functions, or a slide rule, close at hand. In many cases the slide rule is a very convenient substitute for a book of tables. It is, however, much more than that, because by means of a few simple adjustments the actual calculations can be carried through and the result obtained. One has only to learn to read the scales, how to move the slide and indicator, and how to set them accurately, in order to be able to perform long and otherwise difficult

*Any errors were almost certainly introduced by me, Mike Markowski (`mm@udel.edu`), when I typed and scanned this in from one of the manuals that came with my N4-ES slide rule.

calculations.

When people have difficulty in learning to use a slide rule, usually it is not because the instrument is difficult to use. The reason is likely to be that they do not understand the mathematics on which the instrument is based, or the formulas they are trying to evaluate. Some slide rule manuals contain relatively extensive explanations of the theory underlying the operations. Some explain in detail how to solve many different types of problems—for example, various cases which arise in solving triangles by trigonometry. In this manual such theory has deliberately been kept to a minimum. It is assumed that the *theory* of exponents, of logarithms, of trigonometry, and of the slide rule is known to the reader, or will be recalled or studied by reference to formal textbooks on these subjects. This is a brief manual on operational technique and is not intended to be a textbook or workbook. Relatively few exercises are included, and the answers of these (to slide rule accuracy) are given immediately so that learning may proceed rapidly, by self-correction if necessary. Any textbook on mathematics which contains problems suitable for slide rule calculation, and their answers, will provide additional practice.

Some of the special scales described in this manual may not be available on your slide rule. All of the illustrations and problems shown can be worked on the slide rule you purchased. However, the special scales simplify the calculations. Pickett Slide Rules are available with all of the special scales shown in this manual.

Form M-15

PART 1. SLIDE RULE OPERATION

INTRODUCTION

The slide rule is a fairly simple tool by means of which answers to involved mathematical problems can be easily obtained. To solve problems easily and with confidence it is necessary to have a clear understanding of the operation of your slide rule. Speed and accuracy will soon reward the user who makes a careful study of the scale arrangements and the manual.

The slide rule consists of three parts: (1) the stator (upper and lower bars); (2) the slide; (3) the cursor or indicator. The scales on the bars and slide are arranged to work together in solving problems. The hairline on the indicator is used to help the eyes in reading the scales and in adjusting the slide.

Each scale is named by a letter (A, B, C, D, L, S, T) or other symbol at the left end.

Figure 1: Need a caption.

The table below shows some of the mathematical operations which can be done easily and quickly with an ordinary slide rule.

OPERATIONS	INVERSE OPERATIONS
Multiplying two or more numbers	Dividing one number by another
Squaring a number	Finding the square root of a number
Cubing a number	Finding the cube root of a number
Finding the sine, cosine, or tangent of an angle	Finding an angler whose sine, cosine, or tangent is known

Various *combinations* of these operations (such as multiplying two numbers and then finding the square root of the result) are also done. Numbers can be added or subtracted with an ordinary slide rule, but it is usually easier to do these operations by arithmetic[1]

In order to use a slide rule, a computer must know: (1) how to read the scales; (2) how to "set" the slide and indicator for each operation to be done; and (3) how to determine the decimal point in the result.

HOW TO READ THE SCALES

The scale labeled C (on the slide) and the scale D (on the stator bar) are used most frequently. These two scales are exactly alike. The total length of these scales has been separated into many smaller parts by fine lines called "graduations."

Some of these lines on the D scale have large numerals (1, 2, 3, etc.) printed just below them. These lines are called *primary* graduations. On the C scale the numerals are printed above the corresponding graduations. A line labeled 1 at the left end is called the *left* index. A line labeled 1 at the right end is called the *right* index.

Next notice that the distance between 1 and 2 on the D scale has been separated into 10 parts by shorter graduation lines. These are the *secondary* graduations.

[1] By putting special scales on a slide rule, these and certain other operations much more difficult than those shown in the above table can be done easily.

Figure 2: Need a caption.

Figure 3: Need a caption.

(On the 10 inch slide rules these lines are labeled with smaller numerals 1, 2, 3, etc. On the 6 inch rules these lines are not labeled.) Each of the spaces between larger numerals 2 and 3, between 3 and 4, and between the other primary graduations is also sub-divided into 10 parts. Numerals are not printed beside these smaller secondary graduations because it would crowd the numerals too much.

When a number is to be located on the D scale, the *first* digit is located by use of the *primary* graduations. The *second* digit is located by use of the *secondary* graduations. Thus when the number 17 is located, the 1 at the left index represents the 1 in 17. The 7th secondary graduation represents the 7. When 54 is to be located, look first for primary graduation 5, and then for secondary graduation 4 in the space immediately to the right.

There are further sub-divisions, or *tertiary graduations,* on all slide rules. The meaning of these graduations is slightly different at different parts of the scale. It is also different on a 6 inch slide rule than on a 10 inch rule. For this reason a separate explanation must be given for each.

Tertiary graduations on 10 inch rules.

The space between each secondary graduation at the left end of the rule (over to primary graduation **2**) is separated into ten parts, but these shortest graduation marks are not numbered. In the middle part of the rule, between the primary graduations **2** and **4**, the smaller spaces between the *secondary* graduations are separated into five parts. Finally, the still smaller spaces between the secondary graduations at the right of **4** are separated into only two parts.

To find 173 on the D scale, look for primary division **1** (the left index), then for secondary division 7 (numbered) then for smaller subdivision 3 (not numbered, but found as the 3rd very short graduation to the right of the longer graduation for 7).

Similarly, 149 is found as the 9th small graduation mark to the right of the 4th secondary graduation mark to the right of primary graduation **1**.

Figure 4: Need a caption.

Figure 5: Need a caption.

Figure 6: Need a caption.

To find 246, look for primary graduation **2**, then for the 4th secondary graduation after it (the 4th long line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are fifths. Since $\frac{3}{5} = \frac{6}{10}$, then the third graduation, marking *three fifths,* is at the same point as *six tenths* would be.

Tertiary graduations on 6 inch rules.

The space between each secondary graduation at the left end of the rule (over to primary graduation 2) is separated into five parts. In the middle of the rule, between the primary graduations 2 and 5, the smaller spaces between the secondary graduations are separated into two parts. Finally, the still smaller spaces between the secondary graduations at the right of 5 are not subdivided.

To find 170 on the D scale, look for the primary division 1 (the left index), then for the secondary division 7 which is the 7th secondary graduation.

To find 146, look for the primary graduation 1, then for the 4th secondary graduation after it (the 4th secondary line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are fifths. Since $3/5 = 6/10$, then the third graduation, marking *three fifths,* is at the same point as *six tenths* would be.

The number 147 would be half of a small space beyond 146. With the aid of the hairline on the runner the position of this number can be located approximately by the eye. The small space is mentally "split" in half.

The number 385 is found by locating primary graduation 3 and then secondary graduation 8 (the 8th long graduation after 3). Following this, one observes that between secondary graduations 8 and 9 there is one short mark. Think of this as the "5 tenths" mark, which represents 385. The location of 383 can be found approximately by mentally "splitting" the space between 380 and 385 into fifths, and estimating where the 3rd "fifths" mark would be placed. It would be just a little to the right of halfway between 380 and 385.

On the scale below are some sample readings.

Figure 7: Need a caption.

A: 195	F: 206
B: 119	G: 465
C: 110	H: 402
D: 101	I: 694
E: 223	J: 987

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 used in writing numbers are called *digits.* One way to describe a number is to tell how many digits are used in writing it. Thus 54 is a "two-digit number," and 1,348,256 is a "seven-digit number." In many computations only the first two or three digits of a number need to be used to get an approximate result which is accurate enough for practical purposes. Usually not more than the first three digits of a number can be "set" on a six inch slide rule scale. In many practical problems this degree of accuracy is sufficient. When greater accuracy is desired, a ten inch rule is generally used.

Multiplication

Numbers that are to be multiplied are called factors. The result is called the product. Thus, in the statement $6 \times 7 = 42$, the numbers 6 and 7 are factors, and 42 is the product.

EXAMPLE: Multiply 2×3

Setting the Scales: Set the left index of the C scale on 2 of the D scale. Find 3 on the C scale, and below it read the product, 6 on the D scale.

Think: The length for 2 plus the length for 3 will be the length for the product.

This length, measured by the D scale, is 6.

EXAMPLE: Multiply 4×2

Setting the Scales: Set the left index of the C scale on 4 of the D scale. Find 2 on the C scale, and below it read the product, 8, on the D scale.

Think: The length for 4 plus the length for 2 will be the length for the product.

This length, measured by the D scale, is 8.

EXAMPLE: Multiply 2.34×36.8

Estimate the result: First note that the result will be roughly the same as 2×40, or 80; that is, there will be two digits to the left of the decimal point. Hence, we can ignore the decimal points for the present and multiply as though the problem was 234×368.

Setting the Scales: Set the left index of the C scale on 234 of the D scale. Find 368 on the C scale and read product 861 on the D scale.

Think: The length for 234 plus the length for 368 will be the length for the product. This length is measured on the D scale. Since we already knew the result was somewhere near 80, the product must be 86.1, approximately.

EXAMPLE: Multiply 28.3×5.46

Note first that the result will be about the same as 30×5, or 150. Note also that if the left index of the C scale is set over 283 on the D scale, and 546 is then found on the C scale, the slide projects so far to the right of the rule that the D scale is no longer below the 546. When this happens, the other index of the C scale must be used. That is, set the *right* index on the C scale over 283 on the D scale. Find the 546 on the C scale and below it read the product on the D scale. The product is 154.5.

These examples illustrate how in simple problems the decimal point can be placed by use of an estimate.

PROBLEMS	ANSWERS
1. 15×3.7	55.5
2. 280×0.34	95.2
3. 753×89.1	67,100
4. 9.54×16.7	159.3
5. 0.215×3.79	0.0815

Division

In mathematics, division is the opposite or *inverse* operation of multiplication. In using a slide rule this means that the process for multiplication is reversed. To help in understanding this statement, set the rule to multiply 2×4 (see page [with this ex.]). Notice the result 8 is found on the D scale under 4 of the C scale. Now to divide 8 by 4 these steps are reversed. First find 8 on the D scale, set 4 on the C scale over it, and read the result 2 on the D scale under the index of the C scale.

Think: From the length for 8 (on the D scale) *subtract* the length for 4 (on the C scale). The length for the difference, read on the D scale, is the result, or quotient.

With this same setting you can read the quotient of $6 \div 3$, or $9 \div 4.5$, and in fact all divisions of one number by another in which the result is 2.

***Rule for Division:* Set the *divisor* (on the C scale) opposite the number to be divided (on the D scale). Read the result, or quotient, on the D scale under the index of the C scale.**

EXAMPLES:

(a) Find $63.4 \div 3.29$. The quotient must be near 20, since $60 \div 3 = 20$. Set indicator on 63.4 of the D scale. Move the slide until 3.29 of the C scale is under the hairline. Read the result 19.27 on the D scale at the C index.

(b) Find $26.4 \div 47.7$. Since 26.4 is near 25, and 47.7 is near 50, the quotient must be roughly $25/50 = \frac{1}{2} = 0.5$. Set 47.7 of C opposite 26.4 of D, using the indicator to aid the eyes. Read 0.553 on the D scale at the C index.

PROBLEMS	ANSWERS
1. $83 \div 7$	11.86
2. $75 \div 92$	0.815
3. $137 \div 513$	0.267
4. $17.3 \div 231$	0.0749
5. $8570 \div .0219$	391,000

Decimal Point Location

In the discussion which follows, it will occasionally be necessary to refer to the number of "digits" and number of "zeros" in some given numbers.

When numbers are greater than 1 the number of *digits* to the left of the decimal point will be counted. Thus 734.05 will be said to have 3 digits. Although as written the number indicates accuracy to *five* digits, only three of these are at the left of the decimal point.

Numbers that are less than 1 may be written as *decimal fractions*.[2] Thus .673, or six-hundred-seventy-three thousandths, is a decimal fraction. Another example is .000465. In this number three zeros are written to show where the decimal point is located. One way to describe such a number is to tell how many zeros are written to the right of the decimal point before the first non-zero digit occurs.

In scientific work a zero is often written to the left of the decimal point, as in 0.00541. This shows that the number in the units' place is definitely 0, and that no digits have been carelessly omitted in writing or printing. The zeros will *not* be counted unless they are (a) at the *right* of the decimal point, (b) before or at the *left* of the first non-zero digit, and (c) are not between other digits. The number 0.000408 will be said to have 3 zeros (that is, the number of zeros between the decimal point and the 4).

In many, perhaps a majority, of the problems met in genuine applications of mathematics to practical affairs, the position of the decimal point in the result can be determined by what is sometimes called "common sense." There is usually only one place for the decimal point in which the answer is "reasonable" for the problem. Thus, if the calculated speed in miles per hour of a powerful new airplane comes out to be 4833, the decimal point clearly belongs between the 3's, since 48 m.p.h. is too small, and 4833 m.p.h. is too large for such a

[2] Only positive real numbers are being considered in this discussion.

plane. In some cases, however, the data are such that the position of the point in the final result is not easy to get by inspection.

Another commonly used method of locating the decimal point is by estimation or approximation. For example, when the slide rule is used to find 133.4×12.4, the scale reading for the result is 1655, and the decimal point is to be determined. By rounding off the factors to 133.0×10.0, one obtains 1330 by mental arithmetic. The result would be somewhat greater than this but certainly contains four digits on the left of the decimal point. The answer, therefore, must be 1655.

In scientific work numbers are often expressed in *standard form.* For example, 428 can be written 4.28×10^2, and 0.00395 can be written as 3.95×10^-3. When a number is written in standard form it always has two factors. The first factor has one digit (not a zero) on the left of the decimal point, and usually other digits on the right of the decimal point. The other factor is a power of 10 which places the decimal point in its true position if the indicated multiplication is carried out. In many types of problems this method of writing numbers simplifies the calculation and the location of the decimal point.

When written in standard form, the exponent of 10 may be called the "characteristic." It is the characteristic of the logarithm of the number to base 10. The characteristic may be either a positive or a negative number. Although the rule below appears long, in actual practice it may be used with great ease.

***Rule.* To express a number in standard form:**

(a) place a decimal point at the right of the first non-zero digit.[3]

(b) start at the right of the first non-zero digit in the original number and count the digits and zeros passed over in reaching the decimal point. The result of the count is the numerical value of the characteristic, or exponent of 10. If the original decimal point is toward the right, the characteristic is *positive* (+). If the original decimal point is toward the left, the characteristic is *negative* (–). Indicate that the result of (a) is to be multiplied by 10 with the exponent thus determined in (b).

EXAMPLES:

[3]In using this rule, "first" is to be counted from the left; thus, in 3246, the digit 3 is "first."

Number	Number in standard form.	Characteristic
(a) 5,790,000	5.79×10^6	6
(b) 0.000283	2.83×10^{-4}	-4
(c) 44	4.4×10^1	1
(d) 0.623	6.23×10^{-1}	-1
(e) 8.15	8.15×10^0	0
(f) 461,328	4.61328×10^5	5
(g) 0.0000005371	5.371×10^{-7}	-7
(h) 0.0306	3.06×10^{-2}	-2
(i) 80.07	8.007×10^1	1

If a number given in standard form is to be written in "ordinary" form, the digits should be copied, and then starting at the right of the first digit the number of places indicated by the exponent should be counted, supplying zeros as necessary, and the point put down. If the exponent is positive, the count is towards the right; if negative, the count is towards the left. This converse application of the rule may be verified by studying the examples given above.

Consider now the calculation of 5,790,000 $\times$ 0.000283. From examples (a) and (b) above, this can be written as $5.79 \times 10^6 \times 2.83 \times 10^{-4}$, or by changing order and combining the exponents of 10, as $5.79 \times 2.83 \times 10^2$. Then since 5.79 is near 6, and 2.83 is near 3, the product of these two factors is known to be near 18. The multiplication by use of the C and D scales shows it to be about 16.39, or $1.639 \times 10^3 = 1639$. If, however, one has

$5,790,000 \div 0.000283$, the use of standard form yields

$$\frac{5.79 \times 10^6}{2.83 \times 10^{-4}} = 2.04 \times 10^{6-(-4)} = 2.04 \times 10^{10}.$$

In scientific work the result would be left in this form, but for popular consumption it would be written as 20,4000,000,000. The general rule is as follows.

Rule. **To determine the decimal point, first express the numbers in standard form. Carry out the indicated operations of multiplication or division, using the laws of exponents[4] to combine the exponents until a single power of 10 is indicated. If desired, write out the resulting number, using the final exponent of 10 to determine how far, and in what direction, the decimal point in the coefficient should be moved.**

[4] See any textbook on elementary algebra. The theory of exponents and the rules of operation with signed numbers are both involved in a complete treatment of this topic. In this manual it is assumed that the reader is familiar with this theory.

Continued Products

Sometimes the product of three or more numbers must be found. These "continued" products are easy to get on the slide rule.

EXAMPLE: Multiply $38.2 \times 1.65 \times 1.89$.

Estimate the result as follows: $40 \times 1 \times 10 = 400$. The result should be, very roughly, 400.

Setting the Scales: Set the left index of the C scale over 382 on the D scale. Find 165 on the C scale, and *set the hairline* on the indicator on it.[5] Move the index on the slide under the hairline. In this example if the *left* index is placed under the hairline, then 89 on the C scale falls outside the D scale. Therefore move the *right* index under the hairline. Move the hairline to 89 on the C scale and read the result (561) under it on the D scale.

Below is a *general rule* for continued products: $a \times b \times c \times d \times e \ldots$

Set hairline of indicator at a on D scale.
Move index of C scale under hairline.
Move hairline over b on the C scale.
Move index of C scale under hairline.
Move hairline over c on the C scale.
Move index of C scale under hairline.

Continue moving hairline and index alternately until all numbers have been set.

Read result under the hairline on the D scale.

PROBLEMS	ANSWERS
1. $2.9 \times 3.4 \times 7.5$	73.9
2. $17.3 \times 43 \times 9.2$	6,840
3. $343 \times 91.5 \times 0.00532$	167
4. $19 \times 407 \times 0.0021$	16.24
5. $13.5 \times 709 \times 0.567 \times 0.97$	5260

Combined Multiplication and Division

Many problems call for both multiplication and division.

EXAMPLE: $\dfrac{42 \times 37}{65}$.

First, set the division of 42 by 65; that is, set 65 on the C scale opposite 42 on the D scale.[6] Move the hairline on indicator to 37 on the C scale. Read the

[5] The product of 382×165 could now be read under the hairline on the D scale, but this is not necessary.

[6] The quotient, .646, need not be read.

result on the D scale under the hairline. Since the fraction $\frac{42}{65}$ is about equal to $\frac{2}{3}$, the result is about two-thirds of 37, or 23.9.

Example: $\dfrac{273 \times 548}{692 \times 344}$

Set 692 on the C scale opposite 273 on the D scale. Move the hairline to 548 on the C scale. Move the slide to set 344 on the C scale under the hairline. Read the result .628 on the D scale under the C index.

In general, to do computations of the type $\dfrac{a \times c \times e \times g \cdots}{b \times d \times f \times h \cdots}$, set the rule to divide the first factor in the numerator a by the first factor in the denominator b, move the hairline to the next factor in the numerator c; move the slide to set next factor in denominator, d, under the hairline. Continue moving hairline and slide alternately for other factors (e, f, g, h, *etc.*). Read the result on the D scale. If there is one more factor in the numerator than in the denominator, the result is under the hairline. If the number of factors in the numerator and denominator is the same, the result is under the C index. Sometimes the slide must be moved so that index replaces the other.[7]

Example: $\dfrac{2.2 \times 2.4}{8.4}$

If the rule is set to divide 2.2 by 8.4, the hairline cannot be set over 2.4 of the C scale and at the same time remain on the rule. Therefore the hairline is moved to the C index (opposite 262 on the D scale) and the slide is moved end for end to the right (so that the *left* index falls under the hairline and over 262 on the D scale). Then the hairline is moved to 2.4 on the C scale and the result .63 is read on the D scale.

If the number of factors in the numerator exceeds the number in the denominator by more than one, the numbers may be grouped, as shown below. After the value of the *group* is worked out, it may be multiplied by the other factors in the usual manner.

$$\left(\frac{a \times b \times c}{m \times n}\right) \times d \times e$$

[7] This statement assumes that up to this point only the C and D scales are being used. Later sections will describe how this operation may be avoided by the use of other scales.

PROBLEMS	ANSWERS
1. $\dfrac{27 \times 43}{19}$	61.1
2. $\dfrac{5.17 \times 1.25 \times 9.33}{4.3 \times 6.77}$	2.07
3. $\dfrac{842 \times 2.41 \times 173}{567 \times 11.52}$	53.7
4. $\dfrac{1590 \times 3.64 \times 0.763}{4.39 \times 930}$	1.081
5. $\dfrac{0.0237 \times 3970 \times 32 \times 6.28}{0.00029 \times 186000}$	351
6. $\dfrac{231 \times 58.6 \times 4930}{182.5 \times 3770}$	97.0
7. $\dfrac{875 \times 1414 \times 2.01}{661 \times 35.9}$	104.8
8. $\dfrac{558 \times 1145 \times 633 \times 809}{417 \times 757 \times 354}$	2930
9. $\dfrac{0.691 \times 34.7 \times 0.0561}{91,500}$	0.0000147 or 1.47×10^{-5}
10. $\dfrac{19.45 \times 7.86 \times 361 \times 64.4}{32.6 \times 9.74}$	11,190

Proportion

Problems in proportion are very easy to solve. First notice that when the index of the C scale is opposite 2 on the D scale, the ratio 1:2 or $\frac{1}{2}$ is at the *same time* set for all other opposite graduations; that is, 2:4, or 3:6, or 2.5:5, or 3.2:6.4, etc. It is true in general that for any setting the numbers for *all pairs of opposite graduations have the same ratio.* Suppose one of the terms of a proportion is unknown. The proportion can be written as $\frac{a}{b} = \frac{c}{x}$, where a, b, and c, are known numbers and x is to be found.

Rule: **Set a on the C scale opposite b on the D scale. Under c on the C scale read x on the D scale.**

EXAMPLE: find x if $\frac{3}{4} = \frac{5}{x}$.

Set 3 on C opposite 4 on D. Under 5 on C read 6.67 on D.

The proportion above could also be written $\frac{b}{a} = \frac{x}{c}$, or "inverted," and exactly the same rule could be used. Moreover, if C and D are interchanged in the above rule, it will still hold if "under" is replaced by "over." It then reads as follows:

***Rule:* In solving proportions, keep in mind that the position of the numbers as set on the scales is the same as it is in the proportion written in the form $\frac{a}{b} = \frac{c}{d}$.**

Proportions can also be solved *algebraically.* Then $\frac{a}{b} = \frac{c}{d}$ becomes $x = \frac{bc}{a}$, and this may be computed as combined multiplication and division.

PROBLEMS	ANSWERS
1. $\frac{x}{42.5} = \frac{13.2}{1.87}$	300.
2. $\frac{90.5}{x} = \frac{3.42}{1.54}$	40.7
3. $\frac{43.6}{89.2} = \frac{x}{2550}$	1247.
4. $\frac{0.063}{0.51} = \frac{34.1}{x}$	276.
5. $\frac{19}{91} = \frac{13}{x}$	65.7

PART 2. USE OF CERTAIN SPECIAL SCALES

THE CI AND DI SCALES

It should be understood that the use of the CI and DI scales does not increase the power of the instrument to solve problems. In the hands of an experienced computer, however, these scales are used to reduce the number of settings or to avoid the awkwardness of certain settings. In this way the speed can be increased and errors minimized.

The CI scale on the slide is a C scale which *increases from right to left.* It may be used for finding reciprocals. When any number is set under the hairline on the C scale its reciprocal is found under the hairline on the CI scale, and conversely.

EXAMPLES:

(a) Find 1/2.4. Set 2.4 on C. Read .417 directly above on CI.

(b) Find 1/60.5. Set 60.5 on C. Read .0165 directly above on CI. Or, set 60.5 on CI, read .0165 directly below on C.

The CI scale is useful in replacing a division by a multiplication. Since $\frac{a}{b} = a \times 1/b$, any division may be done by multiplying the numerator (or dividend) by the reciprocal of the denominator (or divisor). This process may often be used to avoid settings in which the slide projects far outside the rule.

EXAMPLES:

(a) Find $13.6 \div 87.5$. Consider this as $13.6 \times 1./87.5$. Set left index of the C scale on 13.6 of the D scale. Move hairline to 87.5 on the CI scale. Read the result, .155, on the D scale.

(b) Find $72.4 \div 1.15$. Consider this as $72.4 \times 1/1.15$. Set right index of the C scale on 72.4 of the D scale. Move hairline to 1.15 on the CI scale. Read 63.0 under the hairline on the D scale.

An important use of the CI scale occurs in problems of the following type.

EXAMPLE: Find $\frac{13.6}{4.13 \times 2.79}$ This is the same as $\frac{13.6 \times 1/2.79}{4.13}$. Set 4.13 on the C scale opposite 13.6 on the D scale. Move hairline to 2.79 on the CI scale, and read the result, 1.180, on the D scale.

By use of the CI scale, factors may be transferred from the numerator to the denominator of a fraction (or vice-versa) in order to make the settings more readily. Also, it is sometimes easier to get $a \times b$ by setting the hairline on a, pulling b on the CI scale under the hairline, and reading the result on D scale under the index.

The DI scale (inverted D scale) below the D scale corresponds to the CI scale on the slide. Thus the D and DI scale scales together represent reciprocals. The DI scale has several important uses, of which the following is representative. Expressions of the type 1/X, where X is some complicated expression or formula, may be computed by first finding the value of X. If the result for X falls on D, then 1/X may be read under the hairline of DI.

EXAMPLE:

(a) Find $\frac{1}{0.265 \times 138}$. Multiply 0.265×138 using the C and D scales. Read the reciprocal .0273 under the hairline on the DI scale. Or set the hairline on 265 of the DI scale, pull 138 of the C scale under the hairline, and read the result on the D scale under the left index of the C scale. This is equivalent to writing the expression as $\frac{(1/.265)}{138}$.

PROBLEMS	ANSWERS
1. $\frac{1}{7}$	.143
2. $\frac{1}{35.2}$	.0284
3. $\frac{1}{.1795}$	5.57
4. $\frac{1}{6430}$	.0001555
5. $\frac{1}{\pi}$	.318
6. $\frac{1}{.00417}$	240

THE CF/π AND DF/π SCALES

It should be understood that the use of the CF and DF scales does not increase the power of the instrument to solve problems. In the hands of an experienced computer, however, these scales are used to reduce the number of certain settings. In this way the speed can be increased and errors minimized.

When π on the C scale is opposite the right index of the D scale, about half the slide projects beyond the rule. If this part were cut off and used to fill in the opening at the left end, the result would be a "folded" C scale, or CF scale. Such a scale is printed on top of the slide. It begins at π and the setting of C and D is automatically set on CF and DF. Thus if 3 on C is opposite 2 on D, then 3 on CF is also opposite 2 on DF. The CF and DF scales can be used for multiplication and division in exactly the same way as the C and D scales. the most important use of the CF and DF scales is to avoid resetting the slide. If a setting of the indicator cannot be made on the C or D scale, it can be made on the CF or DF scale.

Examples:

(a) Find 19.2×6.38. Set left index of C on 19.2 of D. Note that 6.38 on C falls outside the D scale. Hence, move the indicator to 6.38 on the CF scale, and read the result 122.5 on the DF scale. Or set the index of CF on 19.2 of DF. Move indicator to 6.38 on CF and read 122.5 on DF.

(b) Find $\frac{8.39 \times 9.65}{5.72}$ Set 5.72 on C opposite 8.39 on D. When the indicator is set on any number N on D, the reading on DF is Nπ. This can be symbolized as (DF) $= \pi$(D). Then (D) $= \frac{(DF)}{\pi}$. This leads to the following simple rule.

***Rule:* If the diameter of a circle is set on D, the circumference may be read immediately on DF, and conversely.**

Examples:

(a) Find 5.6π. Set indicator over 5.6 on D. Read 17.6 under hairline on DF.

(b) Find $8/\pi$. Set indicator over 8 on DF. Read 2.55 under hairline on D.

(c) Find the circumference of a circle whose diameter is 7.2. Set indicator on 7.2 of D. Read 22.6 on DF.

(d) Find the diameter of a circle whose circumference is 121. Set indicator on 121 of DF. Read 38.5 on D.

Finally, these scales are useful in changing radians to degrees and conversely. Since π radians = 180 degrees, the relationship may be written as a proportion $\frac{r}{d} = \frac{\pi}{180}$, or $\frac{r}{\pi} = \frac{d}{180}$.

***Rule:* Set 180 of C opposite π on D. To convert radians to degrees, move indicator to r (the number of radians) on DF, read d (the number of degrees) on CF; to convert degrees to radians, move indicator to d on CF, read r on DF.**

There are also other convenient settings as suggested by the proportion. Thus one can set the ratio $\pi/180$ on the CF and DF scales and find the result from the C and D scales.

EXAMPLES:

(a) The numbers 1, 2, and 7.64 are the measures of three angles in radians. Convert to degrees. Set 180 of C on π of D. Move indicator to 1 on DF, read 57.3^o on CF. Move indicator over 2 of DF, read 114.6°. Move indicator to 7.64 of DF. Read 437° on CF.

(b) Convert 36° and 83.2° to radians. Use the same setting as in (a) above. Locate 36 on CF. Read 0.628 radians on DF. Locate 83.2 on CF. Read 1.45 radians on DF.

PROBLEMS	ANSWERS
1. 1.414×7.79	11.02
2. 2.14×57.6	123.3
3. $\frac{84.5\times7.59}{36.8}$	17.43
4. $2.65 \times \pi$	8.33
5. $\frac{.1955\times23.7}{50.7\times\pi}$	.0291

THE CIF SCALE

Like the other special scales the CIF scale does not increase the power of the instrument to solve problems. It is used to reduce the number of settings or to avoid the awkwardness of certain settings. In this way the speed can be increased and errors minimized.

The CIF scale is a folded CI scale. Its relationship to the CF and DF scales is the same as the relation of the CI scale to the C and D scales.

EXAMPLES:

(a) Find 68.2×1.43. Set the indicator on 68.2 of the D scale. Observe that if the left index is moved to the hairline the slide will project far to the right. Hence merely move 14.3 on CI under the hairline and read the result 97.5 on D at the C index.

(b) Find $2.07 \times 8.4 \times 16.1$. Set indicator 2.07 on C. Move slide until 8.4 on CI is under the hairline. Move hairline to 16.1 on C. Read 280 on D under hairline. Or, set the index of CF on 8.4 of DF. Move indicator to 16.1 on CF, then move slide until 2.07 on CIF is under the hairline. Read 280 on DF above the index of CF. Or set to 16.1 on opposite 8.4 on D. Move indicator to 2.07 on C, and read 280 on D. Although several other methods are possible, the first method given is preferable.

THE CF/M AND DF/M SCALES

The scales CF/M and DF/M are a special feature of PICKETT DUAL BASE LOGLOG SLIDE RULES. Later sections (see Part 4) will explain how the Log

Log scales are used. For the present, we only need to say that on DUAL BASE rules, logarithms to base 10 may be read from the ordinary C or D scales, and "Natural" logarithms (to base e) may be read directly (with the same setting of the indicator) from these special CF/M and DF/M scales.

The same CF/M and DF/M scales are folded at $1/\mathrm{M} = 2.30$, where the modulus $\mathrm{M}{=}\log_{10} e$. The symbol /M is used to distinguish these scales from the ordinary CF and DF scales which are folded π. The index 1 of CF/M and DF/M is about two-thirds of the length of the rule from the left end.

Although these scales have a special purpose, they may be used in ordinary multiplication, division, and other calculations in exactly the same way that the CD/π and DF/π scales are used. This means, in effect, that PICKETT DUAL BASE LOG LOG SLIDE RULES not only have ordinary C and D scales on both sides of the rule, but also have a set of folded scales on each side.

Examples: (a) 19.2×6.38. Set the left index of C on 19.2 of D. Note that 6.38 falls outside the D scale. Hence, move the indicator to 6.38 on the CF/M scale, and read the result 122.5 on the DF/M scale. Observe that the index of the CF/M scale is automatically set at 19.2 of the DF/M scale.

(b) Find $\frac{8.39\times 9.65}{5.72}$ Set 5.72 on C opposite 8.39 on D. The indicator cannot be moved to 9.65 of C, but it can be moved to this setting on CF/M and the result, 14.15, read on the DF/M scale. Or the entire calculation may be done on the CF/M and DF/M scales.

THE $\surd$ SCALES: Square Roots and Squares

When a number is multiplied by itself the result is called the *square* of the number. Thus 25 or 5×5 is the square of 5. The factor 5 is called the *square root* of 25. Similarly, since $12.25 = 3.5 \times 3.5$, the number 12.25 is called the square of 3.5; also 3.5 is called the square root of 12.25. Squares and square roots are easily found on a slide rule.

Square Root. Just below the D scale is another scale marked with the square root symbol, $\surd$.

***Rule.* The square root of any number located on the D scale is found directly below it on the $\surd$ scale.**

Examples: Find $\sqrt{4}$. Place the hairline indicator over 4 on the D scale. The square root, 2, is read directly below. Similarly, the square root of 9 (or $\sqrt{9}$) is 3, found on the $\surd$ scale directly below the 9 on the D scale.

Reading the Scales. The square root scale directly below the D scale is an enlargement of the D scale itself. The D scale has been "stretched" to double its former length. Because of this the square root scale seems to be cut off or to end with the square root of 10, which is about 3.16. To find the square root of numbers greater than 10 the bottom $\surd$ scale is used. This is really the rest of the stretched D scale. The small figure 2 near the left end is placed beside the mark for 3.2, and the number 4 is found nearly two inches farther to the right. In fact, if 16 is located on the D scale, the square root of 16, or 4, is directly below it on the *bottom scale* of the rule.

In general, the square root of a number between 1 and 10 is found on the upper square root scale. The square root of a number between 10 and 100 is found on the lower square root scale. If the number has an odd number of digits or zeros (1, 3, 5, 7, ...), the upper $\sqrt{\ }$ scale is used. If the number has an even number of digits or zeros (2, 4, 6, 8, ...), the lower $\sqrt{\ }$ scale is used. The first three (or in some cases even four) figures of a number may be set on the D scale, and the first three (or four) figures of the square root are read directly from the proper square root scale.

The table below shows the number of digits or zeros in the number N and its square root.

ZEROS					or	DIGITS				
	U L	U L	U L	U	L	U L	U L	U L	U L	
N	7 or 6	5 or 4	3 or 2	1	0	1 or 2	3 or 4	5 or 6	7 or 8	etc.
$\sqrt{N}$	3	2	1	0	0	1	2	3	4	etc.

The above table is reproduced on some models of Pickett Slide Rules.

This shows that numbers from 1 up to 100 have one digit in the square root; numbers from 100 up to 10,000 have two digits in the square root, etc. Numbers which are less than 1 and have, for example, either two or three zeros, have only one zero in the square root. Thus $\sqrt{0.004 = 0.0632}$, and $\sqrt{0.0004 = 0.02}$.

Examples:

(a) Find $\sqrt{248}$. Set the hairline on 248 of the D scale. This number has 3 (an *odd* number) digits. Therefore the figures in the square root are read from the upper $\sqrt{\ }$ scale as 1575. The result has 2 digits, and is 15.75 approximately.

(b) Find $\sqrt{563000}$. Set the hairline on 563 of the D scale. The number has 6 (an *even* number) digits. Read the figures of the square root on the bottom scale as 75. The square root has 3 digits and is 750 approximately.

(c) Find $\sqrt{.00001362}$. Set the hairline on 1362 of the D scale. The number of *zeros* is 4 (an *even* number). Read the figures 369 on the bottom scale. The result has 2 zeros, and is .00369.

***Squaring* is the opposite of finding the square root. Locate the number on the proper $\sqrt{\ }$ scale and with the aid of the hairline read the square on the D scale.**

Examples:

(a) Find $(1.73)^2$ or 1.73×1.73. Locate 1.73on the $\sqrt{\ }$ scale. On the D scale find the approximate square 3.

(b) Find $(62800)^2$. Locate 628 on the $\sqrt{\ }$ scale. Find 394 above it on the D scale. The number has 5 digits. Hence the square has either 9 or 10 digits. Since, however, 628 was located on the lower of the $\sqrt{\ }$ scales, the square has the *even* number of digits, or 10. The result is 3,940,000,000.

(c) Find $(.000254)^2$. On the D scale read 645 above the 254 of the $\sqrt{\ }$ scale. The number has 3 zeros. Since 254 was located on the upper of the $\sqrt{\ }$ scales, the square has the odd number of digits, or 7. The result is 0.0000000645.

PROBLEMS	ANSWERS
1. $\sqrt{7.3}$	2.7
2. $\sqrt{73}$	8.54
3. $\sqrt{841}$	29
4. $\sqrt{0.062}$	0.249
5. $\sqrt{0.00000094}$	0.00097
6. $(3.95)^2$	15.6
7. $(48.2)^2$	2320
8. $(0.087)^2$	0.00757
9. $(0.00284)^2$	0.00000807
10. $(635000)^2$	4.03×10^{11}

THE $\sqrt[3]{\ }$ SCALES: Cube Roots and Cubes

At the top of the rule there is a cube root scale marked $\sqrt[3]{\ }$. It is a D scale which has been stretched to three times its former length, and then cut into three parts which are printed one below the other.

Rule. **The cube root of any number on the D scale is found directly above it on the $\sqrt[3]{\ }$ scales.**

Example: Find the $\sqrt[3]{8}$. Place the hairline of the indicator over the 8 on the D scale. The cube root, 2, is read above on the upper $\sqrt[3]{\ }$ scale.

Reading the scale: To find the cube root of any number between 0 and 10 the upper $\sqrt[3]{\ }$ scale is used. To find the cube root of a number between 10 and 100 the middle $\sqrt[3]{\ }$ scales is used. To find the cube root of a number between 100 to 1000 the lower $\sqrt[3]{\ }$ scale is used.

In general to decide which part of the $\sqrt[3]{\ }$ scale to use in locating the root, mark off the digits in groups of three starting from the decimal point. If the left group contains one digit, the upper $\sqrt[3]{\ }$ scale is used; if there are two digits in the left group, the middle $\sqrt[3]{\ }$ scale is used; if there are three digits, the lower $\sqrt[3]{\ }$ is used. Thus, the roots of numbers containing 1, 4, 7, . . . digits are located on the upper $\sqrt[3]{\ }$ scale; numbers containing 2, 5, 8, . . . digits are located on the middle $\sqrt[3]{\ }$ scale; numbers containing 3, 6, 9 . . . digits are located on the lower $\sqrt[3]{\ }$ scale. The corresponding number of digits or zeros in the cube roots are shown in the table below and whether the upper, middle or lower section of the $\sqrt[3]{}$ scale should be used.

ZEROS					or	DIGITS			
	U M L	U M L	U M L	U M	L	U M L	U M L	U M L	U M L
N	11, 10, 9	8, 7, 6	5, 4, 3	2, 1	0	1, 2, 3	4, 5, 6	7, 8, 9	10, 11, 12
$\sqrt[3]{N}$	3	2	1	0	0	1	2	3	4

The above table is reproduced on some models of Pickett Slide Rules.

EXAMPLES:

(a) Find $\sqrt[3]{6.4}$. Set hairline over 64 on the D scale. Read 1.857 on the upper $\sqrt[3]{}$ scale.

(b) Find $\sqrt[3]{64}$. Set hairline over 64 on the D scale. Read 4 on the middle $\sqrt[3]{}$ scale.

(c) Find $\sqrt[3]{640}$. Set hairline over 64 on the D scale. Read 8.62 on the lower $\sqrt[3]{}$ scale.

(d) Find $\sqrt[3]{6,400}$. Set hairline over 64 on the D scale. Read 18.57 on the upper $\sqrt[3]{}$ scale.

(e) Find $\sqrt[3]{64,000}$. Set hairline over 64 on the D scale. Read 40 on the middle $\sqrt[3]{}$ scale.

(f) Find $\sqrt[3]{0.0064}$. Set hairline over 64 on the D scale. Read 0.1857 on the upper $\sqrt[3]{}$ scale.

(g) Find $\sqrt[3]{0.064}$. Set hairline over 64 on the D scale. Read 0.4 on the middle $\sqrt[3]{}$ scale.

If the number is expressed in standard form it can either be written in ordinary form or the cube root can be found by the following rule.

Rule. **Make the exponent of 10 a multiple of three, and locate the number on the D scale. Read the result on the $\sqrt[3]{}$ scale and multiply this result by 10 to an exponent which is 1/3 the former exponent of 10.**

EXAMPLES: Find the cube root of 6.9×10^3. Place the hairline over 6.9 on the D scale and read 1.904 on the upper $\sqrt[3]{}$ scale. Thus the desired cube root is 1.904×10^1. Find the cube root of 4.85×10^7. Express the number as 48.5×10^6 and place the hairline of the indicator over 48.5 on the D scale. Read 3.65 on the middle $\sqrt[3]{}$ scale. Thus the desired cube root is 3.65×10^2 or 365. Find the cube root of 1.33×10^{-4}. Express the number as 133×10^{-6} and place the hairline over 133 on the D scale. Read 5.10 on the lower $\sqrt[3]{}$ scale. The required cube root is 5.10×10^{-2}.

Cubes: To find the cube of a number, reverse the process for the cube root. Locate the number on the $\sqrt[3]{}$ scale and read the cube of that number on the D scale.

EXAMPLES:

(a) Find $(1.37)^3$. Set the indicator on 1.37 of the $\sqrt[3]{\ }$ scale. Read 2.57 on the D scale.

(b) Find $(13.7)^3$. The setting is the same as in example (a), but the D scale reading is 2570, or 1000 times the former reading.

(c) Find $(2.9)^3$ and $(29)^3$. When the indicator is on 2.9 of the $\sqrt[3]{\ }$ scale, the D scale reading is 24.4. The result for 29^3 is therefore 24,400.

(d) Find $(6.3)^3$. When the indicator is on 6.3 on the $\sqrt[3]{\ }$ scale, the D scale reading is 250.

PROBLEMS	ANSWERS
1. 2.45^3	14.7
2. 56.1^3	176,600
3. $.738^3$	.402
4. 164.5^3	4,451,000
5. $.0933^3$	.000812
6. $\sqrt[3]{5.3}$	1.744
7. $\sqrt[3]{71}$	4.14
8. $\sqrt[3]{815}$	9.34
9. $\sqrt[3]{.0315}$	.316
10. $\sqrt[3]{525,000}$	80.7
11. $\sqrt[3]{.156}$	.538

The L Scale: LOGARITHMS

The L scale represents logarithms to the base 10, or common logarithms. The logarithm of a number is the exponent to which a given base must be raised to produce the number. For example, Log $10^2 = 2.00$; Log $10^3 = 3.00$, etc. A logarithm consists of two parts. The *characteristic* is the part on the left of the decimal point. The *mantissa* is the decimal fraction part on the right of the decimal point. The L scale is used for finding the mantissa of the logarithm (to the base 10) of any number. The mantissa of the logarithm is the same for any series of digits regardless of the location of the decimal point.

The position of the decimal point in the given number determines the characteristic of the logarithm, and conversely. The following rules apply in determining the characteristic.

1. For 1, and all numbers greater than 1, the characteristic is one less than the number of places to the left of the decimal point in the given number.

2. For numbers smaller than 1, that is for decimal fractions, the characteristic

is negative. Its numerical value is one more than the number of zeros between the decimal point and the first significant figure in the given number.

The application of these rules is illustrated in the following chart:

Digits to Left of Decimal Point		Zeros to Right of Decimal Point[8]
Digits in Number	1 2 3 4 5 6 7 8	Zeros in Number 0 1 2 3 4 5 6 7 8
Characteristic	0 1 2 3 4 5 6 7	Characteristic -1 -2 -3 -4 -5 -6 -7 -8 -9

The method described on page 9 is also easy to use.

***Rule.* Locate the number on the C scale and read the mantissa of its logarithm (to the base 10) on the L scale. Determine the characteristic.**

EXAMPLE: Find the logarithm of 425.

Set the hairline over 425 on the C scale. Read the mantissa of the logarithm (.628) on the L scale. Since the number 425 has 3 digits, the characteristic is 2 and the logarithm is 2.628.

EXAMPLE: Find the logarithm of .000627.

Opposite 627 on the C scale find .797 on the L scale. Since the number has 3 zeros, the characteristic is -4 and the logarithm is -4 + .797 and is usually written 6.797-10.

If the logarithm of a number is known, the number may be found by reversing the above process. The characteristic is ignored until the decimal point is to be placed in the number.

EXAMPLE: Find x, if $\log x = 3.248$.

Set the hairline over 248 on the L scale. Below it read the number 177 on the C scale. Since the characteristic is 3, there are 4 digits in the number, $x = 1770$.

Note that the mantissa of a logarithm is always positive but the characteristic may be either positive or negative. In computations, negative characteristics are troublesome and frequently are a source of error. It is customary to handle the difficulty by not actually combining the negative characteristic and positive mantissa. For example, if the characteristic is -4 and the mantissa is .797, the logarithm may be written 0.797-4. This same number may also be written 6.797-10, or 5.797-9 and in other ways as convenient. In each of these forms if the integral parts are combined, the result is -4. Thus $0 - 4 = -4; 6 - 10 = -4; 5 - 9 = -4$. The form which shows that the number 10 is to be subtracted is the most common.

EXAMPLES:

$$\begin{aligned}\log 1 &= 10.000 - 10\\ \log .4 &= 9.602 - 10\\ \log .0004 &= 6.602 - 10\end{aligned}$$

PROBLEMS:	ANSWERS:
Log 3.26	.513
Log 737	2.866
Log .0194	8.288 − 10
Log 54800	4.739
Log X = 2.052	X = 112.7
Log X = 9.831 − 10	X = .678
Log X = .357	X = 2.28
Log X = 1.598	X = 39.6
Log X = 7.154 − 10	X = .001426

The S, T, and ST Scales: TRIGONOMETRY

The branch of mathematics called *trigonometry* arose historically in connection with the measurement of triangles. However, it now has many other uses in various scientific fields.

Some important ratios may be defined in terms of a right triangle as follows:

Sine of angle A $= \frac{\text{side opposite}}{\text{hypotenuse}}$ (written Sin A $= \frac{a}{h}$);

Cosine of angle A $= \frac{\text{adjacent}}{\text{hypotenuse}}$ (written Cos A $= \frac{b}{h}$);

Tangent of angle A $= \frac{\text{side adjacent}}{\text{hypotenuse}}$ (written Tan A $= \frac{a}{b}$);

Cotangent of angle A $= \frac{\text{side adjacent}}{\text{side opposite}}$ (written Cot A $= \frac{b}{a}$);

Secant of angle A $= \frac{\text{hypotenuse}}{\text{side adjacent}}$ (written Sec A $= \frac{h}{b}$);

Cosecant of angle A $= \frac{\text{hypotenuse}}{\text{side opposite}}$ (written Cosec A $= \frac{h}{a}$);

These ratios are functions of the angle. The definitions may be extended to cover cases in which the angle A is not an interior angle of a right triangle, and hence may be greater than 90 degrees. Note that the sine and cosecant are reciprocals, as are the cosine and secant, and the tangent and cotangent. Therefore,

$$\text{Sin A} = \frac{1}{\text{Cosec A}}, \text{ and Cosec A} = \frac{1}{\text{Sin A}};$$
$$\text{Tan A} = \frac{1}{\text{Cot A}}, \text{ and Cot A} = \frac{1}{\text{Tan A}};$$
$$\text{Cos A} = \frac{1}{\text{Sec A}}, \text{ and Sec A} = \frac{1}{\text{Cos A}}.$$

When the sum of the two angles equals 90°, the angles are *complementary.*

$$\text{Sin A} = \cos(90^\circ - A)$$
$$\text{Cos A} = \sin(90^\circ - A)$$
$$\text{Tan A} = \cot(90^\circ - A)$$
$$\text{Cot A} = \tan(90^\circ - A)$$

When the sum of two angles equals 180°, the angles are *supplementary.*

$$\sin(180^\circ - A) = \text{Sin } A$$
$$\cos(180^\circ - A) = -\text{Cos } A$$
$$\tan(180^\circ - A) = -\text{Tan } A$$

The following laws are applicable to any triangle.

$$A + B + C = 180^\circ$$
$$\text{Law of sines: } \frac{\text{Sin } A}{a} = \frac{\text{Sin } B}{b} = \frac{\text{Sin } C}{c}$$
$$\text{Law of cosines: } a^2 = b^2 + c^2 - 2bc\cos A$$

THE S SCALE: Sines and Cosines

The scale marked S is used in finding the approximate sine or cosine of any angle between 5.7 degrees and 90 degrees. Since $\sin x = \cos(90 - x)$, the same graduations serve for both sines and cosines. Thus, $\sin 6^\circ = cos(90^\circ - 6^\circ) = \cos 84^\circ$. The numbers printed at the right of the longer graduations are read when sines are to be found. Those printed at the left are used when cosines are to be found. On the slide rule, angles are divided decimally instead of into minutes and seconds. Thus $\sin 12.7^\circ$ is represented by the 7th small graduation to the right of the graduation marked 78 | 12.

Sines (or cosines) of all angles on the S scale have no digits or zeros—the decimal point is at the left of figures read from the C (or D) scale.

Rule: **To find the sine of an angle on the S scale, set the hairline on the graduation which represents the angle. (Remember to read sines**

from left to right and the numbers to the right of the graduation are for sines). Read the sine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the sine can also be read on the D scale, and the mantissa of the logarithm of the sine (log sin) may be read on the L scale.

Example:

(1) Find sin $15^\circ\ 30'$ and log sin $15^\circ\ 30'$. Set left index of C scale over left index of D scale. Set hairline on 15.5° (i.e., $15^\circ\ 30'$) on S scale. Read $\sin 15.5^\circ = .267$ on C or D scale. Read mantissa of log sin $15.5^\circ = .427$ on L scale. According to the rule for characteristics of logarithms, this would be $9.427 - 10$.

Rule: **To find the cosine of an angle on the S scale, set the hairline on the graduation which represents the angle. (Remember to read cosines from right to left and the numbers to the left of the graduation are for cosines.) Read the cosine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the cosine can also be read on the D scale, and the mantissa of the cosine (log cos) may be read on the L scale.**

Example:

(1) Find $\cos 42^\circ\ 15'$ and $\log\cos 42^\circ\ 15'$. Set left index of C scale over left index of D scale. Set hairline on $\cos 42.25^\circ$ (i.e., $\cos 42^\circ\ 15'$) on S scale. Read $\cos 42.25^\circ = .740$ on C or D scale. Read mantissa of $\log\cos 42.25^\circ = .869$ on L scale. According to rule for characteristics of logarithms, this would be $9.869 - 10$.

Finding the Angle

If the value of trigonometric ratio is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The value of the ratio is set on the C scale, and the angle itself read on the S scale.

Example:

(a) Given $\sin x = .465$, find x. Set indicator on 465 of C scale, read $x = 27.7^\circ$ on the S scale.

(b) Given $\cos x = .289$, find x. Set indicator on 289 on C scale. Read $x = 73.2^\circ$ on the S scale.

PROBLEMS:	ANSWERS:
1. Sin 9.6°	.167
2. Sin 37.2°	.605
3. Sin 79.0°	.982
4. Cos 12.2°	.977
5. Cos 28.6°	.878
6. Cos 37.2°	.794
7. Cosec 15.8°	3.68

Note: Cosec $\theta = \frac{1}{\sin\ \theta}$;

8. Sec 19.3°	1.060

Note: Sec $\theta = \frac{1}{\cos\ \theta}$;

9. Sin $\theta = .1737$	$\theta = 10°$
10. Sin $\theta = .98$	$\theta = 78°$
11. Sin $\theta = .472$	$\theta = 28.2°$
12. Cos $\theta = .982$	$\theta = 10.8°$
13. Cos $\theta = .317$	$\theta = 71.5°$
14. Cos $\theta = .242$	$\theta = 76°$
15. Sec $\theta = 1.054$ (sec $\theta = \frac{1}{\cos\ \theta}$)	$\theta = 18.7°$
16. Cosec $\theta = 1.765$ (cosec $\theta = \frac{1}{\sin\ \theta}$)	$\theta = 34.5°$
17. $\log\sin 10.4°$	$9.256 - 10$
18. $\log\sin 24.2°$	$9.613 - 10$
19. $\log\cos 14.3°$	$9.986 - 10$
20. $\log\cos 39.7°$	$9.886 - 10$
21. $\log\sin\theta = 9.773 - 10$	$\theta = 36.4°$
22. $\log\sin\theta = 9.985 - 10$	$\theta = 75.°$
23. $\log\cos\theta = 9.321 - 10$	$\theta = 77.9.°$
24. $\log\cos\theta = 9.643 - 10$	$\theta = 63.9.°$

THE T SCALE: Tangents and Cotangents

The T scale, together with the C or CI scales, is used to find the value of the tangent or cotangent of angles between 5.7° and 84.3°. Since $\tan x = \cot(90-x)$, the same graduations serve for both tangents and cotangents. For example, if the indicator is set on the graduation marked 30, the corresponding reading on the C scale is .577, the value of $\tan 30°$. This is also the value of $\cot 60°$, since $\tan 30° = \cot(90° - 30° = \cot(60°)$. Moreover, $\tan x = 1/\cot x$; in other words, the tangent and cotangent of the same angle are reciprocals. Thus for the same setting, the reciprocal of $\cot(60°)$, or $1/.577$, may be read on the CI scale as 1.732. This is the value of $\tan 60$.

A single T scale reading from 5.7° to 45° (left to right) and from 45° to 84.3° (right to left)nwill enable you to make all calculations. In order to provide ease in reading and simplifying the solution of certain problems the T scale on some models of slide rules is doubled. That is, the scale for tangents of angles from 5.7° to 45° is above the line and the scale for angles for 45° to 84.3° is below the line. Check your slide rule and determine the section of the manual that is applicable.

For single T scale

Rule. **Set the angle value on the T scale and read**

(i) tangents of angles from 5.7° **to** 45° **on C,**

(ii) tangents of angles from 45° **to** 84.3° **on CI,**

(iii) cotangents of angles from 45° **to** 84.3° **on C,**

(iv) cotangents of angles from 5.7° **to** 45° **on CI.**

If the slide is set so that the C and D scales coincide, these values may also be read on the D scale. Care must be taken to note that the T scale readings for angles between 45° and 84.3° *increase from right to left.*

In case (i) above, the tangent ratios are all between 0.1 and 1.0; that is, the decimal point is at the left of the number as read from the C scale.

In case (ii), the tangents are greater than 1.0, and the decimal point is placed to the right of the first digit as read from the CI scale. For the cotangent ratios in cases (iii) and (iv) the situation is *reversed.* Cotangents for angles between 45° and 84.3° have the decimal point at the left of the number read from the C scale. For angles between 5.7° and 45° the cotangent is greater than 1 and the decimal point is to the right of the first digit read on the CI scale. These facts may be summarized as follows.

Rule. **If the tangent or cotangent artio is read from the C scale, the decimal point is at the *left* of the first digit read. If the value is read from the CI scale, it is at the *right* of the first digit read.**

EXAMPLES:

(a) Find $\tan x$ and $\cot x$ when $x = 9°50'$. First note that $50' = \dfrac{50}{60}$ of 1 degree $= .83°$, approximately. Hence $9°50' = 9.83°$. Locate $x = 9.83°$ on the T scale. Read $\tan x = .173$ on the C scale, and the read $\cot x = 5.77$ on the CI scale.

(b) Find $\tan x$ and $\cot x$ when $x = 68.6°$. Locate 68.6° on the T scale reading from right from left. Read 255 on the CI scale. Since all angles greater than 45° have tangents greater than 1 (that is, have one digit as defined above), $\tan x = 2.55$. Read $\cot 68.6° = .392$ on the C scale.

Finding the Angle

If the value of the trigonometric ratio is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The value of the ratio is set on the C or CI scale, and the angle itself read on the T scale.

EXAMPLES:

(c) Given $\tan x = .324$, find x. Set 324 on the C scale, read $17.9°$ on the T scale.

(d) Given $\tan x = 2.66$, find x. Set 266 on the CI scale, read $x = 69.4°$ on the T scale.

(e) Given $\cot x = .630$, find x. Set 630 on the C scale, read $x = 57.8°$ on the T scale.

(f) Given $\cot x = 1.865$, find x. Set 1865 on the CI scale, read $28.2°$ on the T scale.

For double T scale

Rule. **Set the angle x on the T scale: (i) above the line if 5.7 $\leq x \leq 45°$, and (ii) below the line if $45° \leq x \leq 84.3°$, and the read the value of the tangent on the C scale, and cotangent on the CI scale.**

In case (i), the decimal point of the tangent is at the left of the first digit read on C. In case (ii), the decimal point of the tangent is at right of the frst digit on C. In case (i), the decimal point of the cotangent is at the right of the first digit read on CI. In case (ii), the decimal point is at the left of the first digit read on CI.

EXAMPLES:

(a) Find $\tan 14.7°$ and $\cot 14.7°$. Set indicator over 14.7 on the upper T scale. Read $\tan 14.7° = 0.262$ on C, and $\cot 14.7° = 3.81$ on CI.

(b) Find $\tan 72.3°$ and $\cot 72.3°$. Set indicator over 72.3 on lower T scale. Read $\tan 72.3° = 3.13$ on C and $\cot 72.3 = 0.319$ on CI.

PROBLEMS:	ANSWERS:
1. $\tan 18.6°$	.337
2. $\tan 66.4°$	2.29
3. $\cot 31.7°$	1.619
4. $\cot 83.85°$	.1078
5. $\tan\theta = 1.173$	$\theta = 49.55°$
6. $\cot\theta = .387$	$\theta = 68.84°$