Math 242 Lab 10
Taylor Polynomials and Series

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Lab Assignment

- Complete ALL Lab Assignment Questions (with codes, computation results, and brief response questions from page 3~4)
- Submit “lastnameLab10.nb”
  and “lastnameLab10.pdf” **(File->Save As→pdf)** on Canvas
- Deadline: **Tomorrow 11:59pm**
- Correct computation results (without codes) are available on Canvas →Files→Lab→Lab_10_Taylor Polynomials and Series→lab10_examples_hints

Series

- Syntax: Series[f[x], {x, a, N}]
- Gives expansion of f[x] in terms of (x-a)^k and output up to (x-a)^N (N-th degree Taylor polynomial, around x=a)

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$
$$= \frac{f(a)}{0!}(x-a)^0 + \frac{f'(a)}{1!}(x-a)^1 + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

- The full Taylor series is when n → Infinity.

  This series equals to f[x] itself when it converges.

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

//Normal

- Add “// Normal” after Series[..] will give us an expression without the Big-O term
- Example: Suppose f[x_] = 1/(Sqrt[x^4 + x^3 + 1]) was defined. Then

```
In[30]:= T3[x_] = Series[f[x], {x, 0, 3}]
```

Out[30]= $1 - \frac{x^3}{2} + O[x]^4$

This “O[x]^4” will cause error when plotting T3[x] .

```
In[31]:= T3[x_] = Series[f[x], {x, 0, 3}] // Normal
```

Out[31]= $1 - \frac{x^3}{2}$

This T3[x] can be plot correctly.

SeriesCoefficient

- Syntax: SeriesCoefficient[f[x], {x, a, N}]
- Gives the coefficient of the (x-a)^N term of the Taylor series of f[x], centered at x=a

In[61]:= `Series[f[x], {x, 0, 10}]`

Out[61]= $x - 2x^2 + 2x^3 - \frac{4x^4}{3} + \frac{2x^5}{3} - \frac{4x^6}{15} + \frac{4x^7}{45} - \frac{8x^8}{315} + \frac{2x^9}{315} - \frac{4x^{10}}{2835} + O[x]^{11}$

In[63]:= `SeriesCoefficient[f[x], {x, 0, 5}]`

Out[63]= $\frac{2}{3}$

SeriesCoefficient

- Syntax: SeriesCoefficient[f[x], {x, a, N}]
- Gives the coefficient of the (x-a)^N term of the Taylor series of f[x], centered at x=a

In[64]:= `Series[f[x], {x, 1, 10}]`

Out[64]= $\frac{1}{e^2} - \frac{x-1}{e^2} + \frac{2\,(x-1)^3}{3\,e^2} - \frac{2\,(x-1)^4}{3\,e^2} + \frac{2\,(x-1)^5}{5\,e^2} - \frac{8\,(x-1)^6}{45\,e^2} + \frac{4\,(x-1)^7}{63\,e^2} - \frac{2\,(x-1)^8}{105\,e^2} + \frac{2\,(x-1)^9}{405\,e^2} - \frac{16\,(x-1)^{10}}{14175\,e^2} + O[x-1]^{11}$

In[65]:= `SeriesCoefficient[f[x], {x, 1, 6}]`

Out[65]= $-\frac{8}{45\,e^2}$

FindSequenceFunction

- Syntax: FindSequenceFunction[(List) , n]
- FindSequenceFunction will try to find the general formula in terms of n, based on the input List (a list of some numbers).

```
In[69]:= FindSequenceFunction[{2, 4, 6, 8, 10}, n]

Out[69]= 2 n

In[70]:= FindSequenceFunction[{1, 4, 9, 16, 25}, n]

Out[70]= n^2

In[71]:= FindSequenceFunction[{2, 4, 6, 8, 10}, k]

Out[71]= 2 k
```

Having Mathematica Try and Determine the General Term Formula

```
In[1]:= Clear[f, x]
        f[x_] = x Exp[-2 x]
        TaylorList = Table[SeriesCoefficient[f[x], {x, 0, n}], {n, 0, 10}]
```

Out[2]= $e^{-2x}\,x$

Out[3]= $\left\{0, 1, -2, 2, -\frac{4}{3}, \frac{2}{3}, -\frac{4}{15}, \frac{4}{45}, -\frac{8}{315}, \frac{2}{315}, -\frac{4}{2835}\right\}$

```
In[4]:= a[n_] = FindSequenceFunction[TaylorList, n]
```

Out[4]= $\dfrac{(-1)^n\,2^{-2+n}}{\text{Pochhammer}[1, -2+n]}$

```
In[5]:= SumConvergence[a[n] * x ^ n, n]
```

Out[5]= True

Recall from last week, this tells us for which x does the infinite sum of a[n]x^n converges.

Having Mathematica Try and Determine the General Term Formula

In[1]:=
```
Clear[f, x]
f[x_] = x Exp[-2 x]
TaylorList = Table[SeriesCoefficient[f[x], {x, 0, n}], {n, 0, 10}]
```

Out[2]= $e^{-2x}\,x$

Out[3]= $\left\{0, 1, -2, 2, -\frac{4}{3}, \frac{2}{3}, -\frac{4}{15}, \frac{4}{45}, -\frac{8}{315}, \frac{2}{315}, \ldots \frac{4}{28\ldots}\right\}$

In[4]:=
```
a[n_] = FindSequenceFunction[TaylorList, n]
```

Out[4]= $\dfrac{(-1)^n\, 2^{-2+n}}{\text{Pochhammer}[1, -2+n]}$

In[5]:=
```
SumConvergence[a[n] * x^n, n]
```

Out[5]= True

Repeat the same steps for Q2-(d), but use f[x_] = **(1 - 2x)^(-5)** and SeriesCoefficient[f[x],{x,**1/6**,n}] SumConvergence[a[n] (x - **1/6**)^n, n]

Wrong

- ClearAll
- cos(1-e^x)
- Findsequencefunction[TaylorList, n]
- Seriescoefficient[f[x], {x, a, n}]
- Sumconvergence[x^n, n]
- SumConvergence[x^n, k]
- SumConvergence[x^k, n]

Correct

- ClearAll[s]
- Clear[s]
- Cos[1-Exp[x]]
- Cos[1-E^x]
- FindSequenceFunction[TaylorList, n]
- SeriesCoefficient[f[x], {x, a, n}]
- SumConvergence[x^n, n]