

# Lab 10 Example and Hints - MATH 242

## FALL 2020

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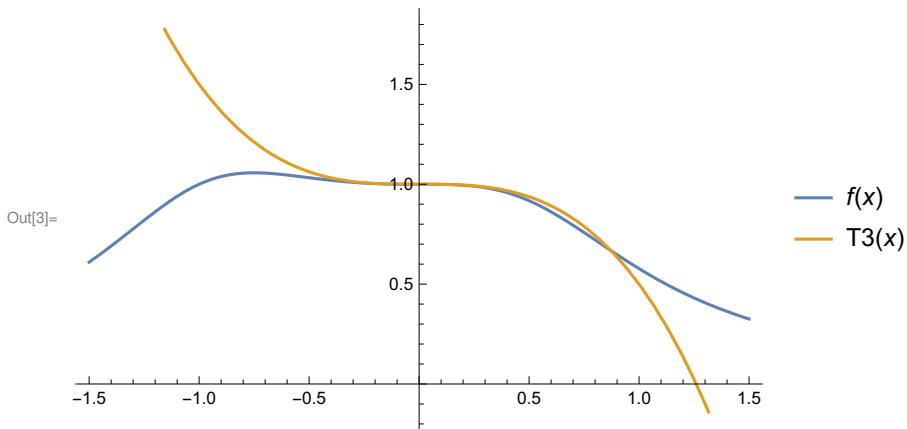
#### Introduction

##### Using Mathematica to Calculate Taylor Polynomials

```
In[1]:= f[x_] = 1 / (Sqrt[x^4 + x^3 + 1])
T3[x_] = Series[f[x], {x, 0, 3}] // Normal
Plot[{f[x], T3[x]}, {x, -1.5, 1.5}, PlotLegends -> "Expressions"]
Out[1]= 
$$\frac{1}{\sqrt{1 + x^3 + x^4}}$$

Out[2]= 
$$1 - \frac{x^3}{2}$$

```

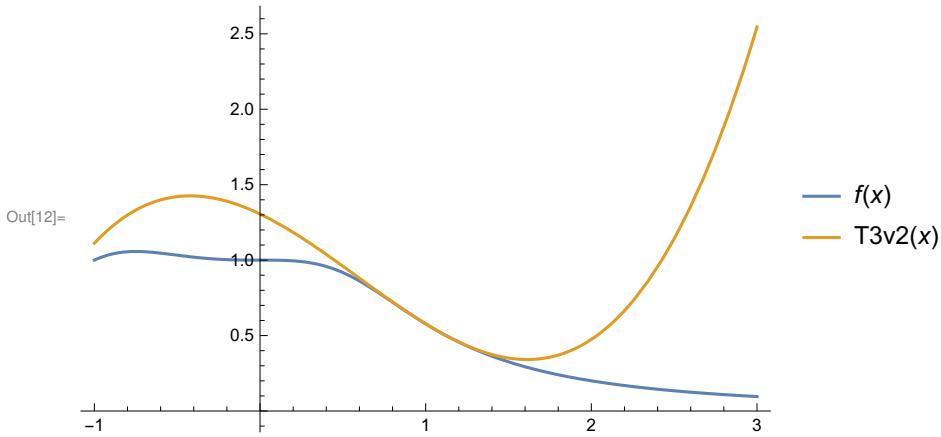


```
In[10]:= f[x_] = 1 / (Sqrt[x^4 + x^3 + 1])
T3v2[x_] = Series[f[x], {x, 1, 3}] // Normal
Plot[{f[x], T3v2[x]}, {x, -1, 3}, PlotLegends -> "Expressions"]

Out[10]= 
$$\frac{1}{\sqrt{1+x^3+x^4}}$$

Out[11]= 
$$\frac{1}{\sqrt{3}} - \frac{7(-1+x)}{6\sqrt{3}} + \frac{13(-1+x)^2}{24\sqrt{3}} + \frac{193(-1+x)^3}{432\sqrt{3}}$$

```



```
In[13]:= f[x_] = 1 / (Sqrt[x^4 + x^3 + 1])
T3[x_] = Series[f[x], {x, -1, 3}] // Normal
T5[x_] = Series[f[x], {x, -1, 5}] // Normal
T7[x_] = Series[f[x], {x, -1, 7}] // Normal
T9[x_] = Series[f[x], {x, -1, 9}] // Normal
Plot[{f[x], T3[x], T5[x], T7[x], T9[x]},
{x, -2, 2}, PlotLegends → "Expressions", PlotRange → {-1, 2}]

Out[13]= 
$$\frac{1}{\sqrt{1+x^3+x^4}}$$

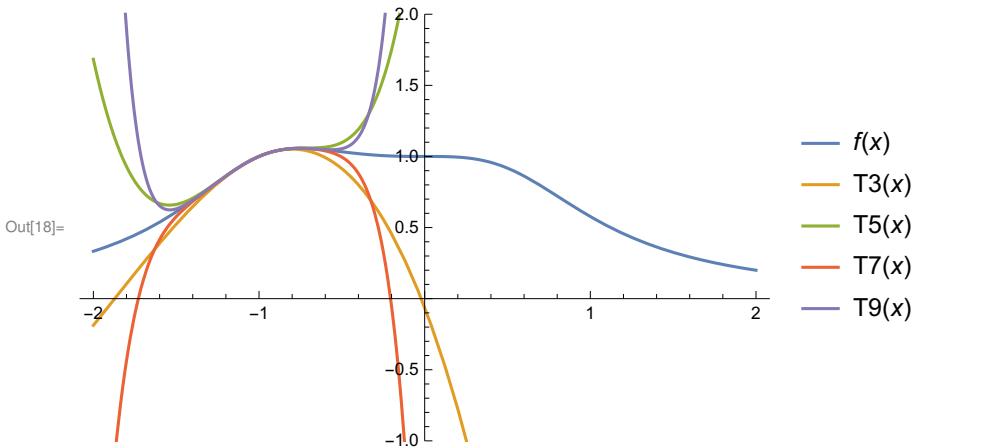

Out[14]= 
$$1 + \frac{1+x}{2} - \frac{9}{8} (1+x)^2 - \frac{7}{16} (1+x)^3$$


Out[15]= 
$$1 + \frac{1+x}{2} - \frac{9}{8} (1+x)^2 - \frac{7}{16} (1+x)^3 + \frac{331}{128} (1+x)^4 + \frac{183}{256} (1+x)^5$$


Out[16]= 
$$1 + \frac{1+x}{2} - \frac{9}{8} (1+x)^2 - \frac{7}{16} (1+x)^3 + \frac{331}{128} (1+x)^4 + \frac{183}{256} (1+x)^5 - \frac{6189 (1+x)^6}{1024} - \frac{2135 (1+x)^7}{2048}$$


Out[17]= 
$$1 + \frac{1+x}{2} - \frac{9}{8} (1+x)^2 - \frac{7}{16} (1+x)^3 + \frac{331}{128} (1+x)^4 + \frac{183}{256} (1+x)^5 - \frac{6189 (1+x)^6}{1024} - \frac{2135 (1+x)^7}{2048} + \frac{481\,395 (1+x)^8}{32\,768} + \frac{64\,811 (1+x)^9}{65\,536}$$

```



## Having Mathematica Try and Determine the General Term Formula

```
In[19]:= Clear[f, x]
f[x_] = x Exp[-2 x]
TaylorList = Table[SeriesCoefficient[f[x], {x, 0, n}], {n, 0, 10}]
Out[20]= 
$$e^{-2x} x$$


Out[21]= 
$$\{0, 1, -2, 2, -\frac{4}{3}, \frac{2}{3}, -\frac{4}{15}, \frac{4}{45}, -\frac{8}{315}, \frac{2}{315}, -\frac{4}{2835}\}$$

```

```
In[22]:= a[n_] = FindSequenceFunction[TaylorList, n]
Out[22]= 
$$\frac{(-1)^n 2^{-2+n}}{\text{Pochhammer}[1, -2+n]}$$

```

```
In[23]:= SumConvergence[a[n] * x^n, n]
Out[23]= True
```

## Assignment Questions

Note : Here's the output for your reference. You may check your answer with mine. But you need to submit the complete codes (input) and output for any credits.

### Q1

(a)

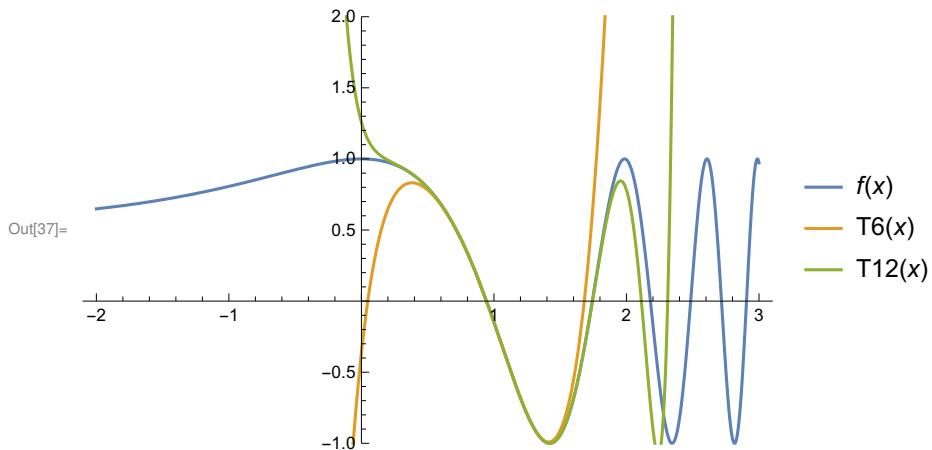
```
Out[53]= Cos[1 - e^x]
Out[54]= Cos[1 - e] + e (-1 + x) Sin[1 - e] +  $\frac{1}{2} (-1 + x)^2 (-e^2 \cos[1 - e] + e \sin[1 - e]) +$ 
 $\frac{1}{24} (-1 + x)^4 (-7 e^2 \cos[1 - e] + e^4 \cos[1 - e] + e \sin[1 - e] - 6 e^3 \sin[1 - e]) +$ 
 $\frac{1}{6} (-1 + x)^3 (-3 e^2 \cos[1 - e] + e \sin[1 - e] - e^3 \sin[1 - e]) + \frac{1}{120} (-1 + x)^5$ 
 $(-15 e^2 \cos[1 - e] + 10 e^4 \cos[1 - e] + e \sin[1 - e] - 25 e^3 \sin[1 - e] + e^5 \sin[1 - e]) +$ 
 $\frac{1}{720} (-1 + x)^6 (-31 e^2 \cos[1 - e] + 65 e^4 \cos[1 - e] - e^6 \cos[1 - e] +$ 
 $e \sin[1 - e] - 90 e^3 \sin[1 - e] + 15 e^5 \sin[1 - e])$ 
```

$$\begin{aligned}
\text{Out}[55]= & \cos[1 - e] + e(-1 + x)\sin[1 - e] + \frac{1}{2}(-1 + x)^2(-e^2\cos[1 - e] + e\sin[1 - e]) + \\
& \frac{1}{6}(-1 + x)^3(-3e^2\cos[1 - e] + e\sin[1 - e] - e^3\sin[1 - e]) + \\
& (-1 + x)^4\left(-\frac{7}{24}e^2\cos[1 - e] + \frac{1}{24}e^4\cos[1 - e] + \frac{1}{24}e\sin[1 - e] - \frac{1}{4}e^3\sin[1 - e]\right) + \\
& (-1 + x)^5\left(-\frac{1}{8}e^2\cos[1 - e] + \frac{1}{12}e^4\cos[1 - e] + \frac{1}{120}e\sin[1 - e] - \frac{5}{24}e^3\sin[1 - e] + \frac{1}{120}e^5\sin[1 - e]\right) + \\
& (-1 + x)^6\left(-\frac{31}{720}e^2\cos[1 - e] + \frac{13}{144}e^4\cos[1 - e] - \frac{1}{720}e^6\cos[1 - e] + \frac{1}{720}e\sin[1 - e] - \frac{1}{8}e^3\sin[1 - e] + \frac{1}{48}e^5\sin[1 - e]\right) + \\
& (-1 + x)^8\left(-\frac{127}{40320}e^2\cos[1 - e] + \frac{27}{640}e^4\cos[1 - e] - \frac{19}{2880}e^6\cos[1 - e] + \frac{e^8\cos[1 - e]}{40320} + \frac{e\sin[1 - e]}{40320} - \frac{23}{960}e^3\sin[1 - e] + \frac{5}{192}e^5\sin[1 - e] - \frac{e^7\sin[1 - e]}{1440}\right) + \\
& (-1 + x)^7\left(-\frac{1}{80}e^2\cos[1 - e] + \frac{5}{72}e^4\cos[1 - e] - \frac{1}{240}e^6\cos[1 - e] + \frac{e\sin[1 - e]}{5040} - \frac{43}{720}e^3\sin[1 - e] + \frac{1}{36}e^5\sin[1 - e] - \frac{e^7\sin[1 - e]}{5040}\right) + \\
& (-1 + x)^9\left(-\frac{17}{24192}e^2\cos[1 - e] + \frac{37}{1728}e^4\cos[1 - e] - \frac{7}{960}e^6\cos[1 - e] + \frac{e^8\cos[1 - e]}{10080} + \frac{e\sin[1 - e]}{362880} - \frac{605}{72576}e^3\sin[1 - e] + \frac{331}{17280}e^5\sin[1 - e] - \frac{11}{8640}e^7\sin[1 - e] + \frac{e^9\sin[1 - e]}{362880}\right) + \\
& (-1 + x)^{10}\left(-\frac{73}{518400}e^2\cos[1 - e] + \frac{6821}{725760}e^4\cos[1 - e] - \frac{1087}{172800}e^6\cos[1 - e] + \frac{5e^8\cos[1 - e]}{24192} - \frac{e^{10}\cos[1 - e]}{3628800} + \frac{e\sin[1 - e]}{3628800} - \frac{311}{120960}e^3\sin[1 - e] + \frac{3}{256}e^5\sin[1 - e] - \frac{7}{4320}e^7\sin[1 - e] + \frac{e^9\sin[1 - e]}{80640}\right) + \\
& (-1 + x)^{12}\left(-\frac{2047}{479001600}e^2\cos[1 - e] + \frac{55591}{43545600}e^4\cos[1 - e] - \frac{30083}{10886400}e^6\cos[1 - e] + \frac{4819}{14515200}e^8\cos[1 - e] - \frac{31}{8709120}e^{10}\cos[1 - e] + \frac{e^{12}\cos[1 - e]}{479001600} + \frac{e\sin[1 - e]}{479001600} - \frac{437}{2419200}e^3\sin[1 - e] + \frac{209}{72576}e^5\sin[1 - e] - \frac{679}{518400}e^7\sin[1 - e] + \frac{e^9\sin[1 - e]}{21504} - \frac{e^{11}\sin[1 - e]}{7257600}\right) + \\
& (-1 + x)^{11}\left(-\frac{31}{1209600}e^2\cos[1 - e] + \frac{265}{72576}e^4\cos[1 - e] - \frac{259}{57600}e^6\cos[1 - e] + \frac{e^8\cos[1 - e]}{3360} - \frac{e^{10}\cos[1 - e]}{725760} + \frac{e\sin[1 - e]}{39916800} - \frac{2591}{3628800}e^3\sin[1 - e] + \frac{2243}{362880}e^5\sin[1 - e] - \frac{277}{172800}e^7\sin[1 - e] + \frac{e^9\sin[1 - e]}{34560} - \frac{e^{11}\sin[1 - e]}{39916800}\right)
\end{aligned}$$

(b)

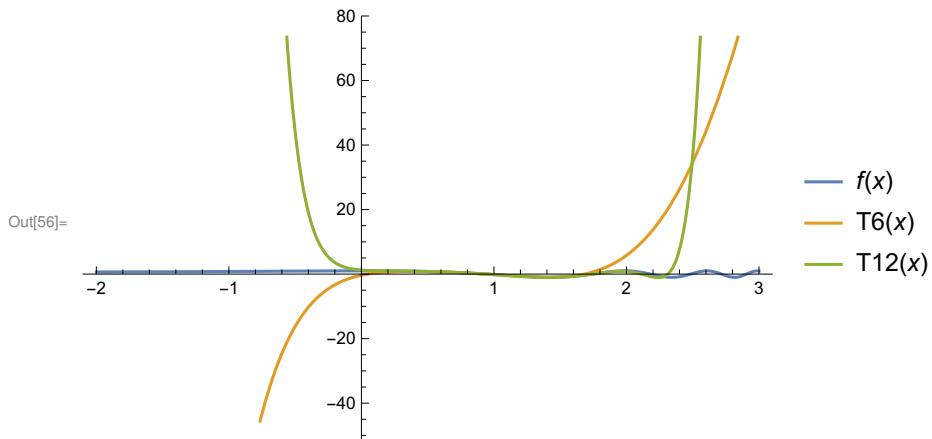
It'll look like the following plot, if you use “PlotRange->{-1,2}” (which will fix the y-axis be -1 to 2)

```
In[37]:= Plot[{f[x], T6[x], T12[x]}, {x, -2, 3},
  PlotLegends → "Expressions", PlotRange → {-1, 2}]
```



It'll look like the following plot, if you do not use “PlotRange” at all. Either way is fine!

```
In[56]:= Plot[{f[x], T6[x], T12[x]}, {x, -2, 3}, PlotLegends → "Expressions"]
```



## Q2

(a)

$$\text{Out}[57]= \frac{1}{(1 - 2x)^5}$$

$$\text{Out}[58]= \frac{243}{32} + \frac{3645}{32} \left(-\frac{1}{6} + x\right) + \frac{32805}{32} \left(-\frac{1}{6} + x\right)^2 + \frac{229635}{32} \left(-\frac{1}{6} + x\right)^3 + \frac{688905}{16} \left(-\frac{1}{6} + x\right)^4$$

$$\text{Out}[59]= -\frac{1}{32 \left(-\frac{1}{2} + x\right)^5}$$

(b)

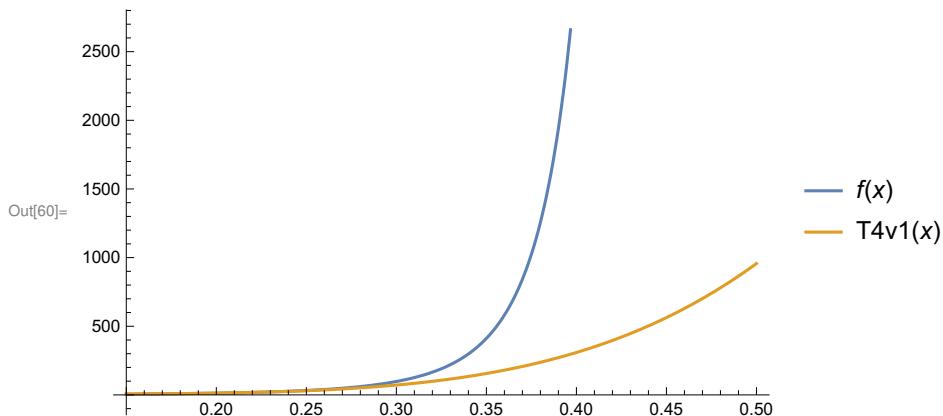
(Brief response, no computation required. You can copy the output from part (a) to answer “What is the output” )

**Hint:** Recall the formula for Taylor series of  $f[x]$  at  $x=a$ . We need  $f[a]$  in the formula.

So we can't get a Taylor series of  $f[x]$  at  $x=a$  if  $f[a]$  is undefined.

(c)

Note: You might want to choose a larger “PlotRange” (which is the range of the y-axis), or just get rid of this option. If you use “PlotRange->{-1,2}” nothing will show.



(d)

**SeriesCoefficient:**

$$\text{Out}[61]= \left\{ \frac{\frac{243}{32}}, \frac{\frac{3645}{32}}, \frac{\frac{32805}{32}}, \frac{\frac{229635}{32}}, \frac{\frac{688905}{16}}, \frac{\frac{3720087}{16}}, \frac{\frac{18600435}{16}}, \frac{\frac{87687765}{16}}, \frac{\frac{789189885}{32}}, \frac{\frac{3419822835}{32}}, \frac{\frac{14363255907}{32}} \right\}$$

**FindSequenceFunction:**

$$\text{Out}[63]= \frac{1}{256} \times 3^{3+n} n (1+n) (2+n) (3+n)$$

**SumConvergence:**

Note: Since the center is at  $x=1/6$  now, the term should be  $a[n] * (x-1/6)^n$  instead of  $a[n]^n x^n$

$$\text{Out}[64]= \text{Abs}\left[-\frac{1}{6} + x\right] < \frac{1}{3}$$

Why is this the largest possible interval of convergence we could hope for?

(brief response)

**Hint:** First, find the endpoints (by hand) of the interval of convergence that we found by SumConvergence.

One of this endpoint is undefined in  $f[x]=(1-2x)^{-5}$ , so the power series representation of  $f[x]$  can't be convergent at this point.

This implies that the interval of convergence must not include this point.

(And we already know that the interval of convergence should be centered at  $x=1/6$  because we're working on the power series that is centered at  $x=1/6$ )