

Lab 9 Example and Hints - MATH 242

FALL 2020

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Introduction

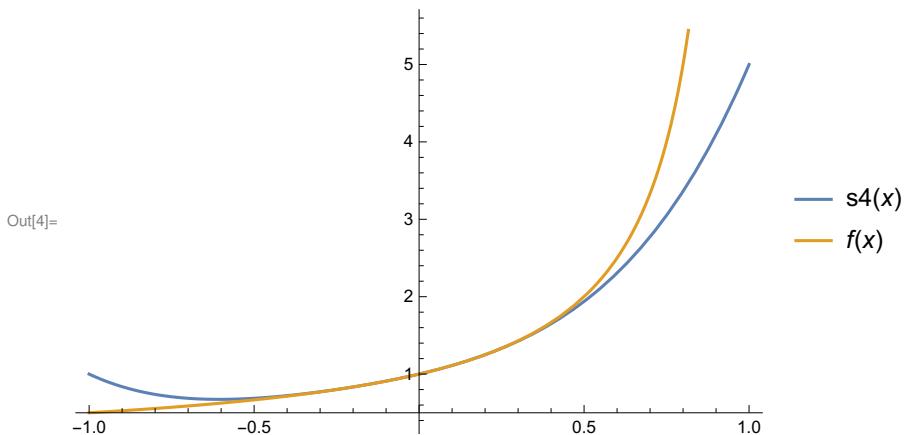
```
In[1]:= SumConvergence[x^n, n]
f[x_] = Sum[x^n, {n, 0, Infinity}]

Out[1]= Abs[x] < 1

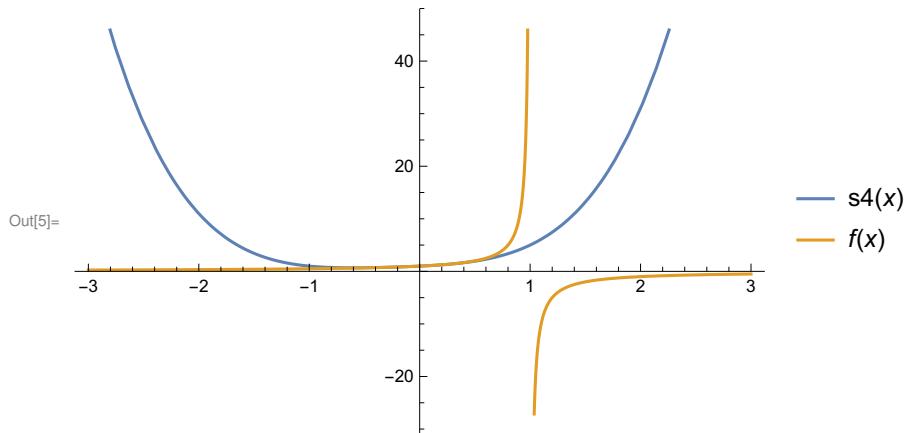
Out[2]= 
$$\frac{1}{1-x}$$


In[3]:= s4[x_] = Sum[x^n, {n, 0, 4}]
Plot[{s4[x], f[x]}, {x, -1, 1}, PlotLegends -> "Expressions"]

Out[3]= 1 + x + x^2 + x^3 + x^4
```

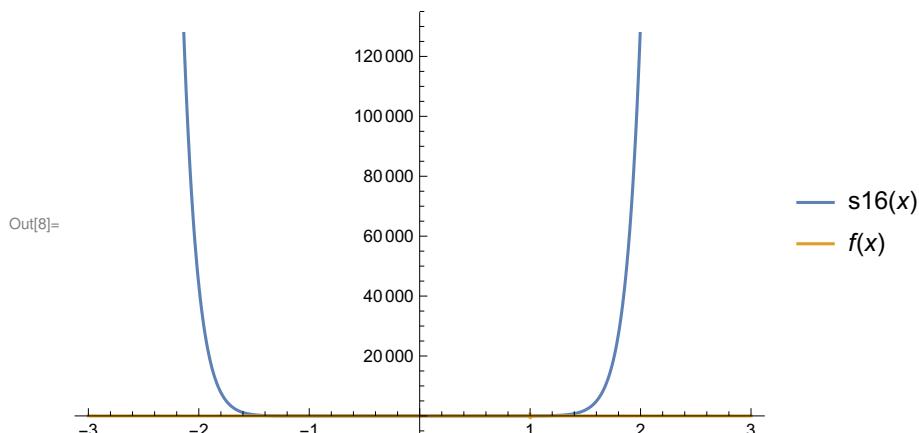
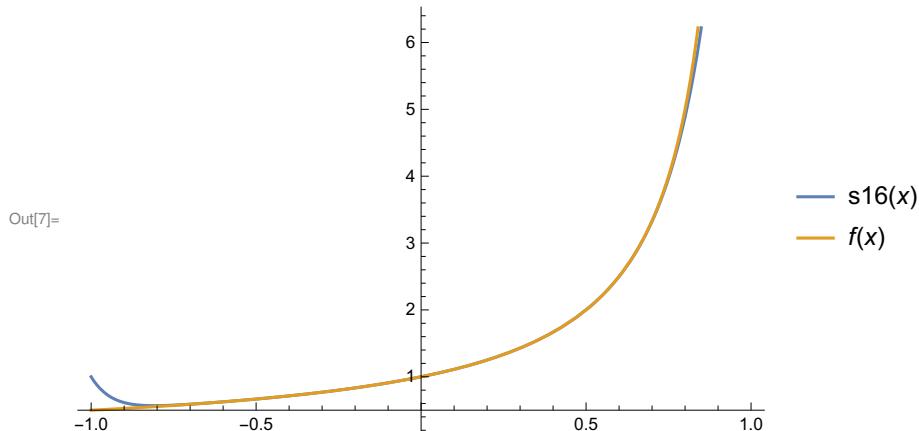


```
In[5]:= Plot[{s4[x], f[x]}, {x, -3, 3}, PlotLegends → "Expressions"]
```



```
In[6]:= s16[x_] = Sum[x^n, {n, 0, 16}]
Plot[{s16[x], f[x]}, {x, -1, 1}, PlotLegends → "Expressions"]
Plot[{s16[x], f[x]}, {x, -3, 3}, PlotLegends → "Expressions"]
```

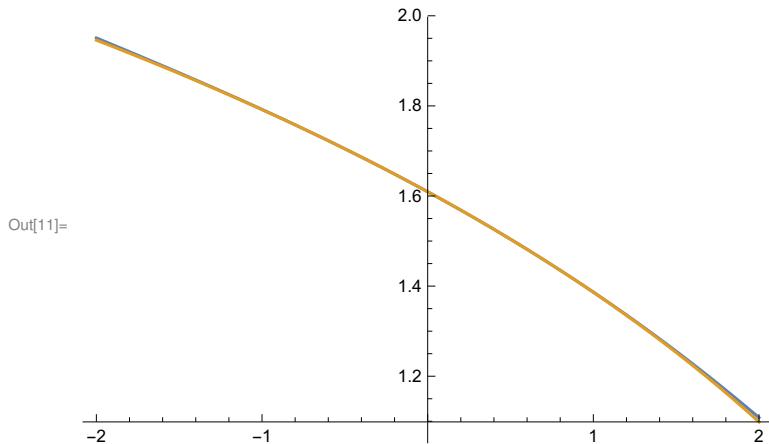
```
Out[6]= 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^10 + x^11 + x^12 + x^13 + x^14 + x^15 + x^16
```



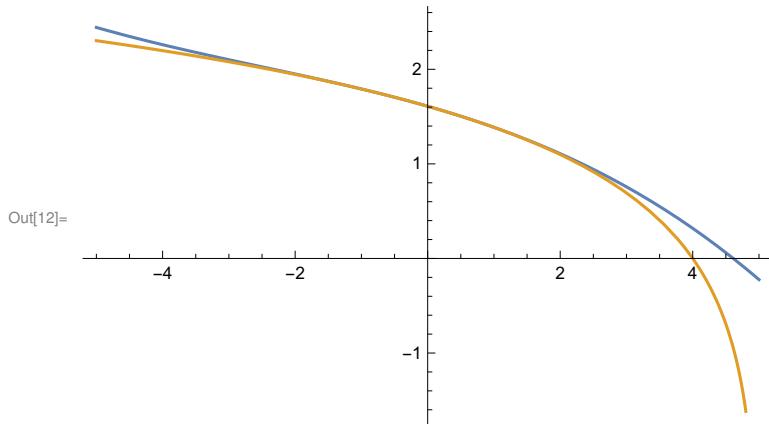
```
In[9]:= Series[Log[5 - x], {x, 0, 5}]
Out[9]= Log[5] -  $\frac{x}{5} - \frac{x^2}{50} - \frac{x^3}{375} - \frac{x^4}{2500} - \frac{x^5}{15625} + O[x]^6$ 
```

```
In[10]:= logSeries[x_] = Log[5] -  $\frac{x}{5} - \frac{x^2}{50} - \frac{x^3}{375}$ 
Plot[{logSeries[x], Log[5 - x]}, {x, -2, 2}]
```

```
Out[10]=  $-\frac{x}{5} - \frac{x^2}{50} - \frac{x^3}{375} + \text{Log}[5]$ 
```



```
In[12]:= Plot[{logSeries[x], Log[5 - x]}, {x, -5, 5}]
```



```
In[13]:= SumConvergence[x^n / (n * 5^n), n]
```

```
Out[13]= Abs[x] < 5 || x == -5
```

```
In[14]:= NumberLinePlot[SumConvergence[x^n / (n * 5^n), n], x]
```



Assignment Questions

Note : Here's the output for your reference. You may check your answer with mine. But you need to submit the complete codes (input) and output for any credits.

Q1

(a)

Out[15]= **True**

Hint: “True” means the series converges for all real number x.

(b)

Out[16]= **BesselJ[1, x]**

(c)

Hint: We can look up the syntax of FunctionDomain by typing the following

In[19]:= **?FunctionDomain**

FunctionDomain[f, x] finds the largest domain of definition of the real function *f* of the variable *x*.

FunctionDomain[f, x, dom] considers *f* to be a function with arguments and values in the domain *dom*.

FunctionDomain[{*funcs*, *vars*, *dom*} finds the

largest domain of definition of the mapping *funcs* of the variables *vars*.

FunctionDomain[{{*funcs*, *cons*}, *vars*, *dom*} finds the domain of *funcs* with

the values of *vars* restricted by constraints *cons*. >

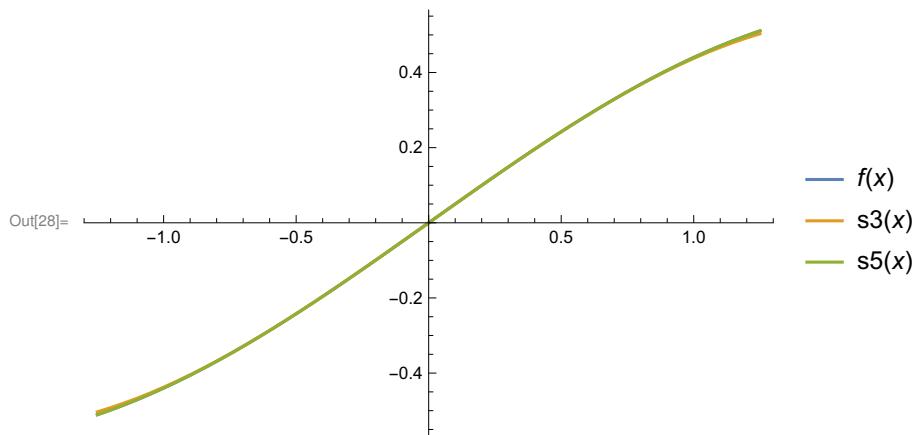
Out[18]= **True**

(d)

Hint: The exponent of x in this series is $2k+1$, so in order to get 3rd degree, we should take k up to 1, but not 3. Similarly, k up to 2 will give the 5th-degree partial sums.

Out[26]=
$$\frac{x}{2} - \frac{x^3}{16}$$

$$\text{Out}[27]= \frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384}$$



(e)

(brief response; no computation required)

Q2

(a)

Note: Remember to use **Log[]**, NOT **ln()**. And use parentheses for $1+x$ and $1-x$.

$$\text{Out}[29]= \text{Log}\left[\frac{1+x}{1-x}\right]$$

$$\text{Out}[30]= -1 < x < 1$$

(b)

$$\text{Out}[31]= 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + O[x]^6$$

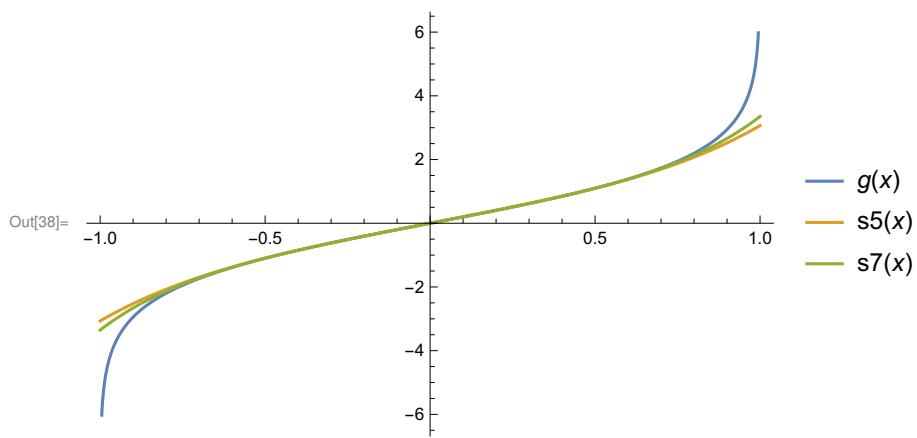
$$\text{Out}[32]= 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7} + O[x]^8$$

(c)

Hint: Define $s5[x_]=$ (copy and paste the first answer in (b) but do not include $O[x]^6$). Similarly define $s7[x_]$.

$$\text{Out}[36]= 2x + \frac{2x^3}{3} + \frac{2x^5}{5}$$

$$\text{Out}[37]= 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7}$$

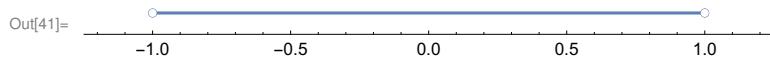


(d)

Hint: One way to write down the positive odd numbers is $2k+1$ for $k=0,1,2,\dots$

(e)

$$\text{Out}[40]= \text{Abs}[x]^2 \leq 1 \&& x^2 \neq 1$$



Hint: The radius of convergence, the interval of convergence and the endpoints can be read from the number line plot.