

Lab 6 Example and Hints - MATH 242

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A first example

```
In[1]:= p[x_] = Apart[(x + 5) / (x^2 + x - 2)]
Integrate[p[x], x]
Out[1]= 
$$\frac{2}{-1+x} - \frac{1}{2+x}$$

Out[2]= 2 Log[1 - x] - Log[2 + x]
```

Polynomial long division

```
In[3]:= q[x_] = Apart[(x^3 + x) / (x - 1)]
Integrate[q[x], x]
Out[3]= 
$$2 + \frac{2}{-1+x} + x + x^2$$

Out[4]= 
$$\frac{1}{6} (-17 + 12x + 3x^2 + 2x^3 + 12 \operatorname{Log}[-1+x])$$

In[5]:= r[x_] = Apart[(x^4 - 2x^2 + 4x + 1) / (x^3 - x^2 - x + 1)]
Integrate[r[x], x]
Out[5]= 
$$1 + \frac{2}{(-1+x)^2} + \frac{1}{-1+x} + x - \frac{1}{1+x}$$

Out[6]= 
$$-\frac{2}{-1+x} + \frac{1}{2} (1+x)^2 + \operatorname{Log}[1-x] - \operatorname{Log}[1+x]
In[7]:= Integrate[(x^4 - 2x^2 + 4x + 1) / (x^3 - x^2 - x + 1), x]
Out[7]= -\frac{2}{-1+x} + \frac{1}{2} (1+x)^2 + \operatorname{Log}[1-x] - \operatorname{Log}[1+x]$$

```

More on repeated roots

```
In[8]:= Solve[{  
    a + b == 0,  
    c - b == 1,  
    8 a + 4 b - c + d == 10,  
    -4 b + 4 c - d + e == 3,  
    16 a - 4 c - e == 36  
},  
{a, b, c, d, e}]  
  
Out[8]= {{a → 2, b → -2, c → -1, d → 1, e → 0}}
```

Assignment Questions

Note : Here's the output for your reference. You may check your answer with mine. But you need to submit the complete codes (input) and output for any credits.

Q1

(essay question)

Hint: Compare the result in the usual math form in page 1 of the handout $2\ln|x-1|$ -....., and the output obtained from the practice code from the section “A first example”

Q2

Note: Apart is required, so the output for each integrals should have both the partial fraction decomposition of the integrand, and the integration result.

$$\text{Out}[9]= \frac{1}{5 x} + \frac{2 (5 x + 2 x^3)}{5 (5 + 5 x^2 + x^4)}$$

$$\text{Out}[10]= \frac{\text{Log}[x]}{5} + \frac{1}{5} \text{Log}[5 + 5 x^2 + x^4]$$

$$\text{Out}[11]= \frac{-2 + x}{(2 + 2 x + x^2)^2} + \frac{x}{2 + 2 x + x^2}$$

$$\text{Out}[12]= \frac{1}{2} \left(-\frac{4+3x}{2+2x+x^2} - 5 \text{ArcTan}[1+x] + \text{Log}[2+2x+x^2] \right)$$

$$\text{Out}[13]= \frac{1}{3(-1+x)} + \frac{-2-x}{3(1+x+x^2)}$$

$$\text{Out}[14]= -\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{1}{3} \text{Log}[1-x] - \frac{1}{6} \text{Log}[1+x+x^2]$$

$$\text{Out}[15]= \frac{1}{16x} - \frac{x}{4(4+x^2)^2} - \frac{x}{16(4+x^2)}$$

$$\text{Out}[16]= \frac{1}{8(4+x^2)} + \frac{\text{Log}[x]}{16} - \frac{1}{32} \text{Log}[4+x^2]$$

$$\text{Out}[17]= -\frac{1}{1+\sqrt{x}} + \frac{1}{x^{3/2}} - \frac{1}{x} + \frac{1}{\sqrt{x}}$$

$$\text{Out}[18]= -\frac{2}{\sqrt{x}} + 2 \text{Log}[1+\sqrt{x}] - \text{Log}[x]$$

Q3

(essay question)