Math 242 Lab 5
More on Numerical Integration

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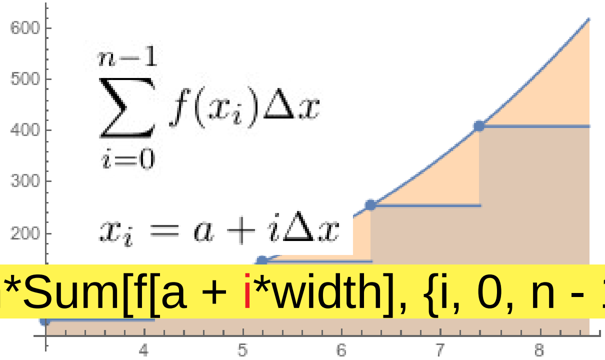
Lab Assignment

- Complete ALL Lab Assignment Questions (with codes, computation results, and essay questions in page 3~4)
- Submit “lastnameLab05.nb” and “lastnameLab05.pdf” **(File->Save As→pdf)** on Canvas
- Deadline: **Tomorrow 11:59pm**
- Correct computation results (without codes) are available on Canvas→Files→Lab→Lab_05_More on Numerical Integration→lab05_examples_hints

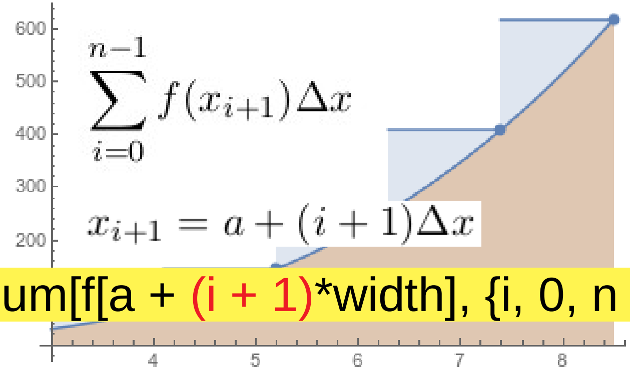
Recall: Four methods from Lab 4

- Left endpoint (rectangle)



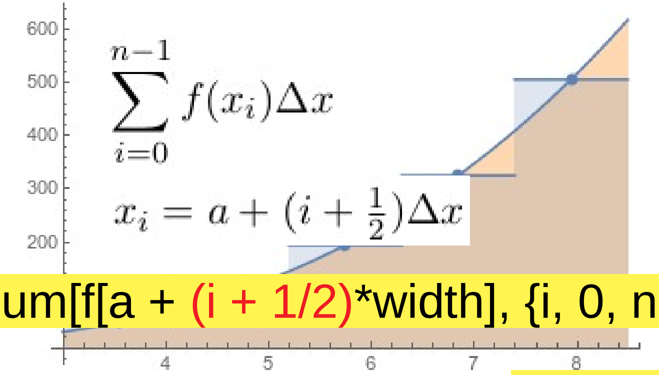
width*Sum[f[a + i*width], {i, 0, n - 1}]//N

- Right endpoint (rectangle)



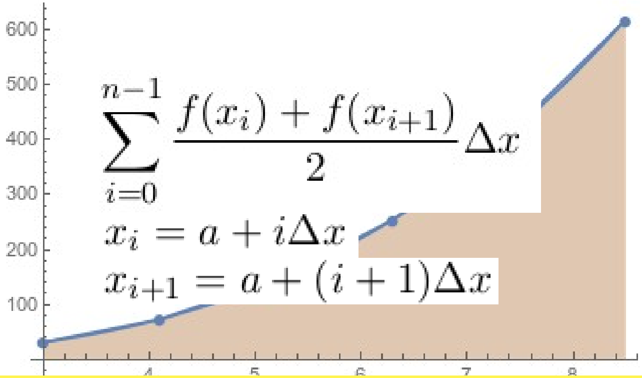
width*Sum[f[a + (i + 1)*width], {i, 0, n - 1}]//N

- Midpoint (rectangle)



width*Sum[f[a + (i + 1/2)*width], {i, 0, n - 1}] // N

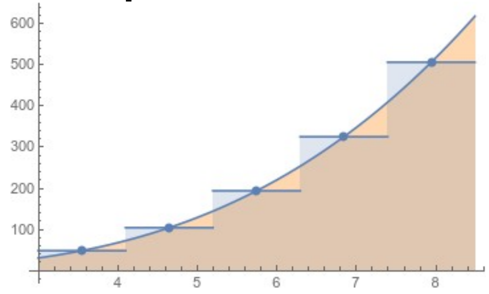
- Trapezoid



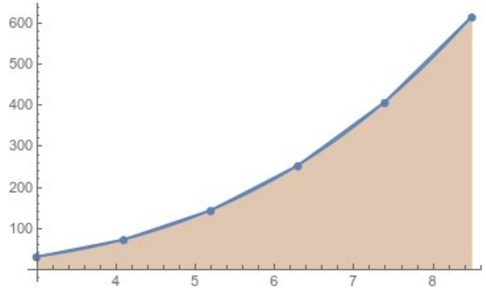
width*Sum[(f[a + i*width] + f[a + (i + 1)*width])/2, {i, 0, n - 1}] // N

Approximate the **curve** by **straight line** segments

- Left endpoint
- Right endpoint
- Midpoint



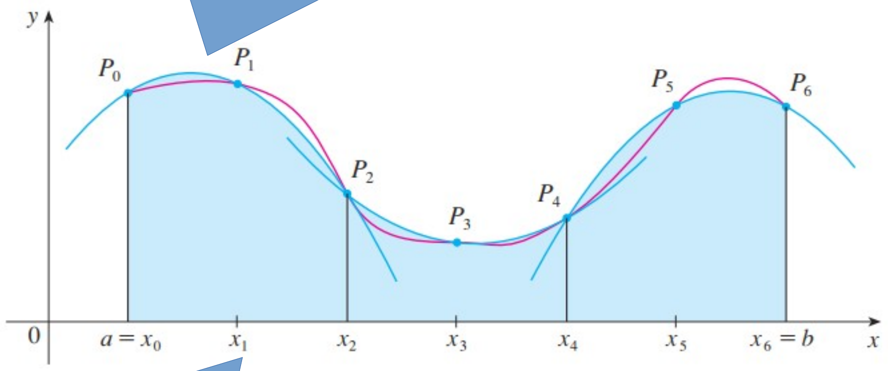
- Trapezoid



Approximate the **curve** by **parabola** segments

- Simpson’s method
  (new introduced in Lab 5)

Red line: the curve f(x)
Blue line: parabolas segments

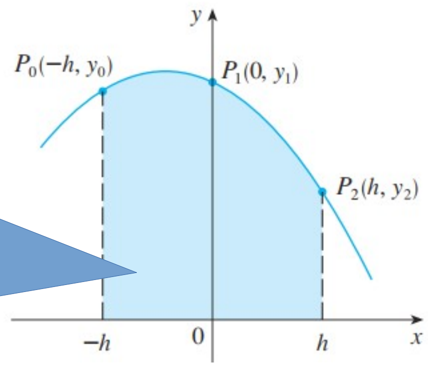


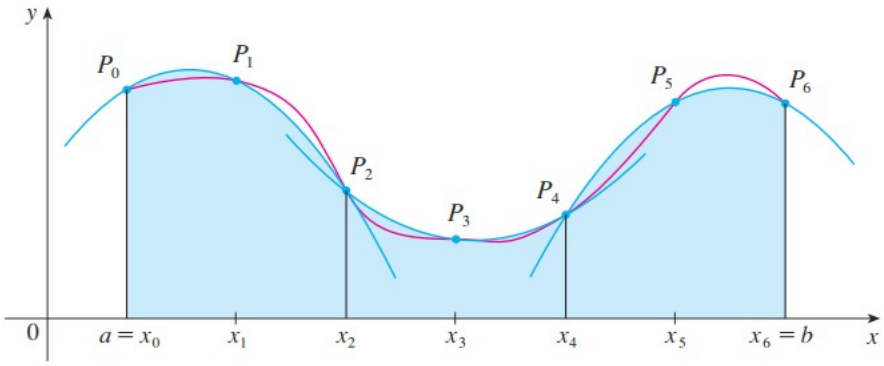
We need three points to determine a parabola, so each small approx. area uses two intervals.
---> n must be an even number!

Simpson’s method

In Sec 7.7 the textbook derived that this **area** is

$$\frac{h}{3}(y_0 + 4y_1 + y_2)$$

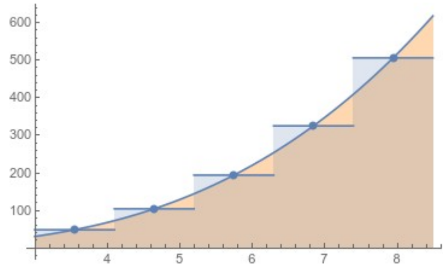




$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(a) + 4f(a+\Delta x) + 2f(a+2\Delta x) + 4f(a+3\Delta x) + 2f(a+4\Delta x) + \dots \right.$$

$$\left. + 2f(b-2\Delta x) + 4f(b-\Delta x) + f(b)\right].$$

- Midpoint:

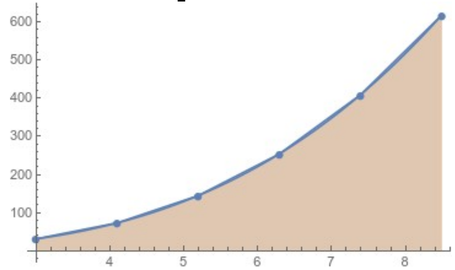


$$\sum_{i=0}^{n-1} f(x_i)\Delta x$$

width*Sum[f[a + (i + 1/2)*width], {i, 0, n - 1}] // N

$$x_i = a + (i + \tfrac{1}{2})\Delta x$$

- Trapezoid:



$$\sum_{i=0}^{n-1} \frac{f(x_i) + f(x_{i+1})}{2}\Delta x$$

$$x_i = a + i\Delta x$$

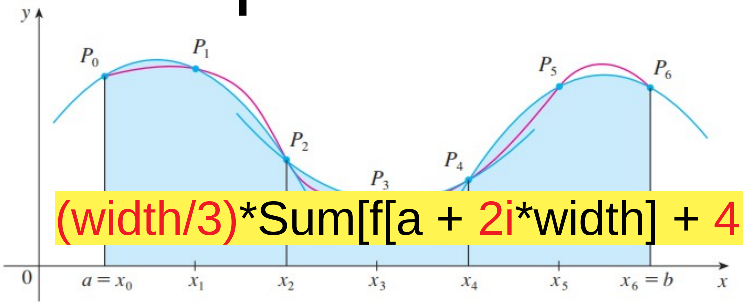
$$x_{i+1} = a + (i+1)\Delta x$$

width*Sum[(f[a + i*width] + f[a + (i + 1)*width])/2, {i, 0, n - 1}] // N

- Simpson's:

$$\frac{\Delta x}{3}\sum_{i=0}^{n/2-1} f(a + 2i\Delta x) + 4f(a + (2i+1)\Delta x) + f(a + (2i+2)\Delta x)$$

n must be even!



(width/3)*Sum[f[a + 2i*width] + 4 f[a + (2i+1)*width] +f[a + (2i +2)*width], {i, 0, n/2 - 1}] // N

Question 1 –
compare Trapezoid & Simpson's

- **Trapezoid method:**

  Repeat Lab 04 Q3(a) and (b), but modify nList={…} and the range of target approximation should be within exact***1.01** and exact***0.99**

Question 1 –
compare Trapezoid & Simpson's

- **Simpson's method:**

  Remember that **n must be even** in Simpson's method. So….
- If you use While Loop:
  - Replace "n=1" by "**n=2**" in the initialization.
  - Replace "n++" by "**n+=2**" or "**n=n+2**" in the While Loop.
  - Replace the code for trapezoid method for Simpson's method.
- If you change n manually:
  - Start with n=2, only use **even number** for your n.
  - Replace the code for trapezoid method for Simpson's method.

Question 2 – Degree of exactness

- For each methods (midpoint, trapezoid, Simpson’s), we want to find the largest value p such that the method can calculate the integral of x^p exactly.
- Start with p=0, compute $\int_0^1 x^p \, dx$

  with exact answer (Integrate[….]) and each approximation method. Stop when approx ≠ exact

Question 2 – Degree of exactness

- For example, you should find midpoint method approximate 1, x perfectly, but sightly off for x^2. That means **the midpoint method is exact for degree 1 polynomial (straight line)**.

Wrong

- e^2x
- exp^2x
- Exp^[2x]
- e(2x)
- Cos[2Pix]
- Cos[2pi x]
- cos(2pix)

Correct

- E^(2x)
- Exp[2x]
- Note: “E” is the number e=2.71828…, and “Exp[x]” is the function e^x.
- Cos[2Pi x]
- Cos[2Pi*x]