# Math 242 Lab 5
# More on Numerical Integration

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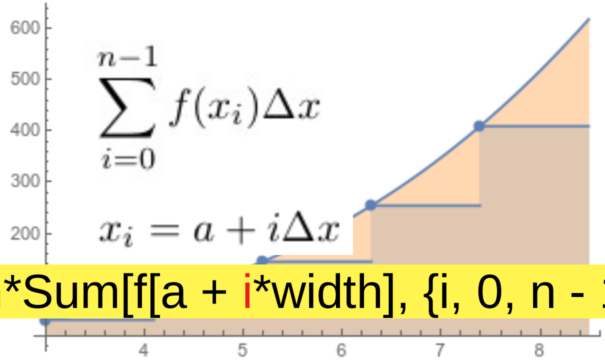
# Lab Assignment

- Complete ALL Lab Assignment Questions (with codes, computation results, and essay questions in page 3~4)
- Submit “lastnameLab05.nb”
  and “lastnameLab05.pdf” **(File->Save As→pdf)** on Canvas
- Deadline: **Tomorrow 11:59pm**
- Correct computation results (without codes) are available on Canvas→Files→Lab→Lab_05_More on Numerical Integration→lab05_examples_hints

# Recall: Four methods from Lab 4

- Left endpoint (rectangle)

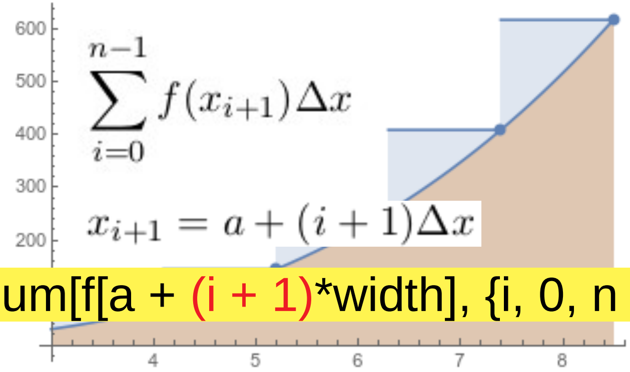


$$\sum_{i=0}^{n-1} f(x_i)\Delta x$$

$$x_i = a + i\Delta x$$

width*Sum[f[a + i*width], {i, 0, n - 1}]//N

- Right endpoint (rectangle)

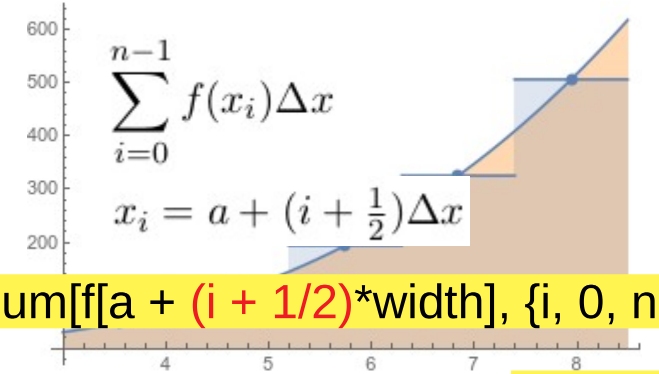


$$\sum_{i=0}^{n-1} f(x_{i+1})\Delta x$$

$$x_{i+1} = a + (i+1)\Delta x$$

width*Sum[f[a + (i + 1)*width], {i, 0, n - 1}]//N

- Midpoint (rectangle)

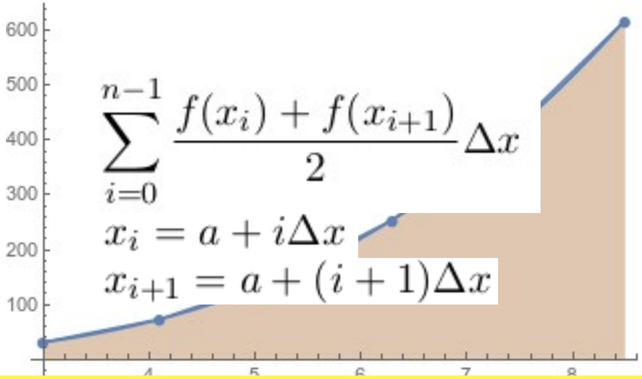


$$\sum_{i=0}^{n-1} f(x_i)\Delta x$$

$$x_i = a + (i + \tfrac{1}{2})\Delta x$$

width*Sum[f[a + (i + 1/2)*width], {i, 0, n - 1}] // N

- Trapezoid



$$\sum_{i=0}^{n-1} \frac{f(x_i) + f(x_{i+1})}{2}\Delta x$$

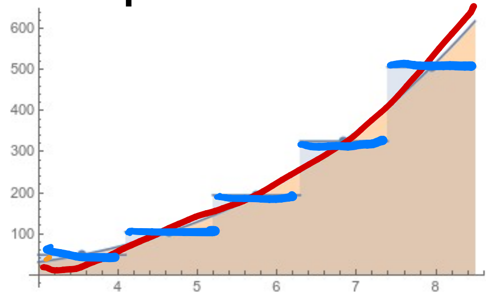
$$x_i = a + i\Delta x$$

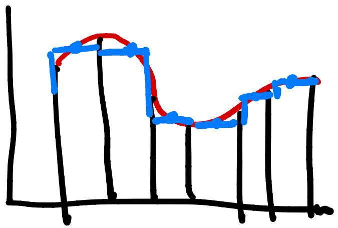
$$x_{i+1} = a + (i+1)\Delta x$$

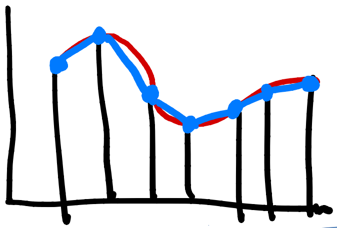
width*Sum[(f[a + i*width] + f[a + (i + 1)*width])/2, {i, 0, n - 1}] // N

Approximate the **curve** by
**straight line** segments

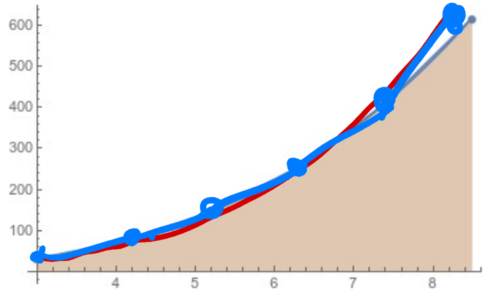
- Left endpoint
- Right endpoint
- Midpoint







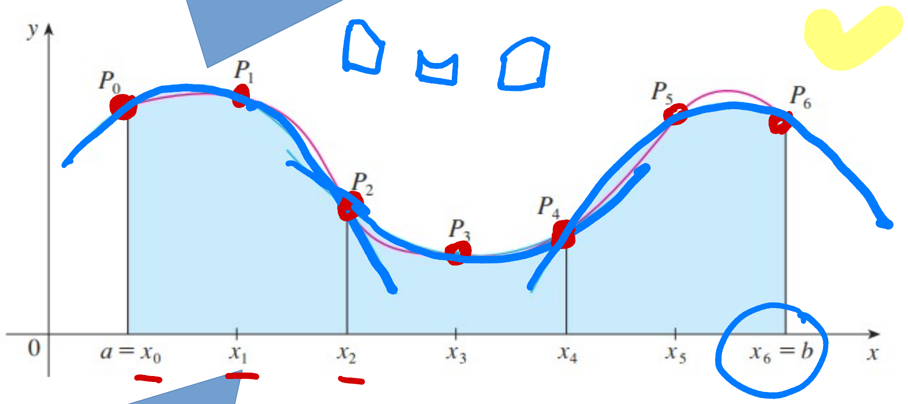
- Trapezoid



Approximate the **curve** by
**parabola** segments

- Simpson’s method
  (new introduced in Lab 5)

Red line: the curve f(x)
Blue line: parabolas segments




We need three points to determine a parabola,
so each small approx. area uses two intervals.
---> n must be an even number!

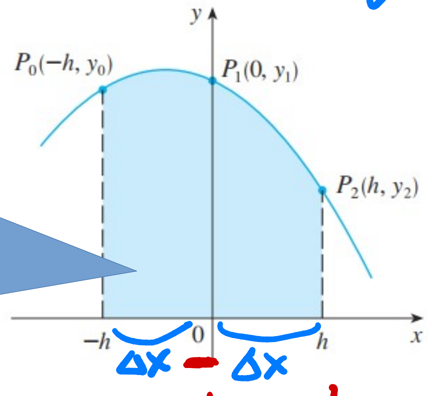
# Simpson's method

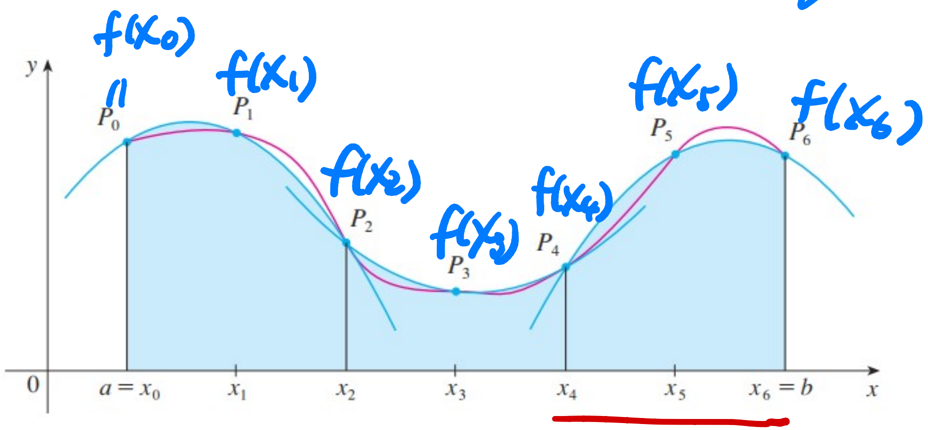
In Sec 7.7 the textbook derived that this **area** is

$\Delta x$

$$\frac{h}{3}(y_0 + 4y_1 + y_2)$$

$f(x_0) = y_0$, $f(x_1) = y_1$, $f(x_2) = y_2$



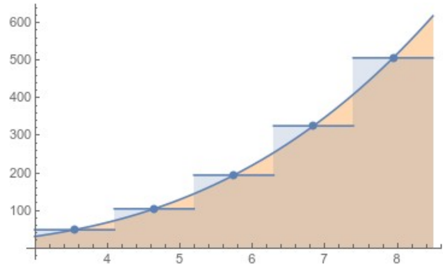




$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}[f(a) + 4f(a+\Delta x) + 2f(a+2\Delta x) + 4f(a+3\Delta x) + 2f(a+4\Delta x) + \ldots$$

$$+2f(b-2\Delta x) + 4f(b-\Delta x) + f(b)].$$

$$= \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right) + \frac{\Delta x}{3}\left(f(x_2) + 4f(x_3) + f(x_4)\right) + \frac{\Delta x}{3}\left(f(x_4) + 4(x_5) + f(x_6)\right)$$

- Midpoint:

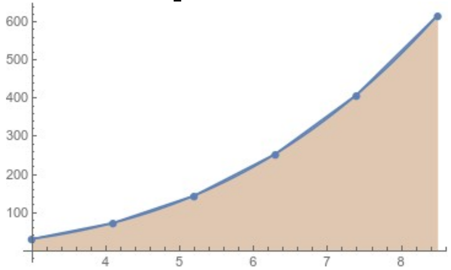



$$\sum_{i=0}^{n-1} f(x_i)\Delta x$$

width*Sum[f[a + (i + 1/2)*width], {i, 0, n - 1}] // N

$$x_i = a + (i + \tfrac{1}{2})\Delta x$$

- Trapezoid:




$$\sum_{i=0}^{n-1} \frac{f(x_i) + f(x_{i+1})}{2}\Delta x$$

$$x_i = a + i\Delta x$$

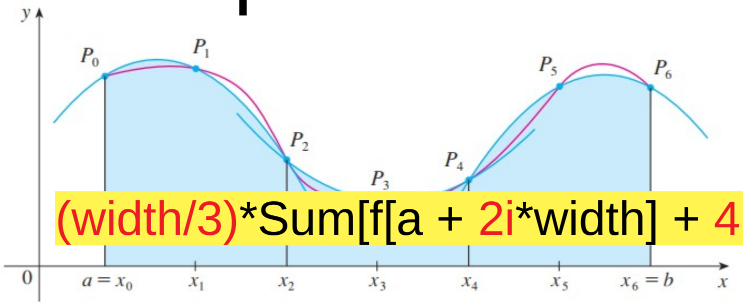
$$x_{i+1} = a + (i+1)\Delta x$$

width*Sum[(f[a + i*width] + f[a + (i + 1)*width])/2, {i, 0, n - 1}] // N

- Simpson's:

$$\frac{\Delta x}{3}\sum_{i=0}^{n/2-1} f(a + 2i\Delta x) + 4f(a + (2i+1)\Delta x) + f(a + (2i+2)\Delta x)$$

n must be even!




(width/3)*Sum[f[a + 2i*width] + 4 f[a + (2i+1)*width] +f[a + (2i +2)*width], {i, 0, n/2 - 1}] // N

# Question 1 –<br>compare Trapezoid & Simpson's

- **Trapezoid method:**

  Repeat Lab 04 Q3(a) and (b), but modify nList={…} and the range of target approximation should be within exact***1.01** and exact***0.99**

# Question 1 – compare Trapezoid & Simpson's

- **Simpson's method:**

  Remember that **n must be even** in Simpson's method. So….

- If you use While Loop:
  - Replace "n=1" by "**n=2**" in the initialization.
  - Replace "n++" by "**n+=2**" or "**n=n+2**" in the While Loop.
  - Replace the code for trapezoid method for Simpson's method.
- If you change n manually:
  - Start with n=2, only use **even number** for your n.
  - Replace the code for trapezoid method for Simpson's method.

# Question 2 – Degree of exactness

- For each methods (midpoint, trapezoid, Simpson's), we want to find the largest value p such that the method can calculate the integral of x^p exactly.
- Start with p=0, compute $\int_0^1 x^p \, dx$

  with exact answer (Integrate[….]) and each approximation method. Stop when approx ≠ exact

$\int_0^1 1 \, dx$ "$x^0$" ? ✓  $\int_0^1 x \, dx$ ? ✓  $\underline{\int_0^1 x^2 \, dx}$ . . .

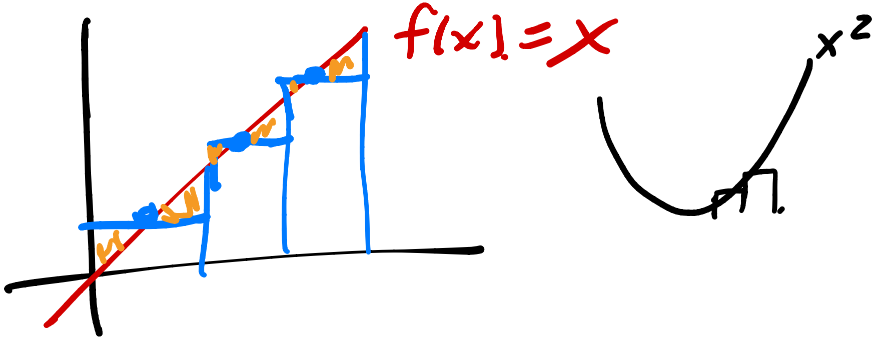
# Question 2 – Degree of exactness

- For example, you should find midpoint method approximate 1, x perfectly, but sightly off for x^2. That means **the midpoint method is exact for degree 1 polynomial (straight line)**.

exact for

deg 1 { mid — 1, x
Trap —

deg 3. (Simp — 1, x, $x^2$, $x^3$.



# Wrong

- e^2x
- exp^2x
- Exp^[2x]
- e(2x)
- Cos[2Pix]
- Cos[2pi x]
- cos(2pix)

# Correct

- E^(2x)
- Exp[2x]
- Note: “E” is the number e=2.71828…, and “Exp[x]” is the function e^x.
- Cos[2Pi x]
- Cos[2Pi*x]