

# Lab 5 Example and Hints - MATH 242

## FALL 2020

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#### Code for the trapezoid method

```
In[]:= f[x_] = 5 * Exp[-x/2] * Sin[x]
a = 0;
b = 8;
n = 10;
width = (b - a) / n;
trap = width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}] // N
Out[]= 5 e-x/2 Sin[x]
Out[]= 3.70389
```

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#### Code for Simpson's method

```
In[]:= f[x_] = 5 * Exp[-x/2] * Sin[x]
a = 0;
b = 8;
n = 10;
width = (b - a) / n;
simp =
  (width/3) * Sum[f[a + 2 i * width] + 4 f[a + (2 i + 1) * width] + f[a + (2 i + 2) * width],
  {i, 0, n/2 - 1}] // N
Out[]= 5 e-x/2 Sin[x]
Out[]= 3.97641
```

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#### Code for the exact solution

```
In[]:= Integrate[f[x], {x, 0, 8}] // N
Out[]= 3.97442
```

It looks like in this case, Simpson's method is more accurate.

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## Degree of exactness

### Midpoint method

```
In[®]:= f[x_] = 1
a = 0;
b = 1;
n = 10;
width = (b - a) / n;
mid = width * Sum[f[a + (i + 1/2) * width], {i, 0, n - 1}] // N

Out[®]= 1

Out[®]= 1.
```

### Trapezoid method

```
In[®]:= trap = width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}] // N

Out[®]= 1.
```

### Simpson's method

```
In[®]:= simp = (width / 3) * Sum[f[a + 2 i * width] +
4 f[a + (2 i + 1) * width] + f[a + (2 i + 2) * width], {i, 0, n/2 - 1}] // N

Out[®]= 1.
```

We see that each approximation is equal to the exact value which is 1.

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## Assignment Questions

Note : Here's the output for your reference. You may check your answer with mine. But you need to submit the complete codes (input) and output for any credits.

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### Q1

Note: Should be almost the same code as Lab 4 Q3a, but you need to modify nList=...

**Trapezoid:**

```
27163.3
13581.6
10451.
4248.05
2544.59
1272.29
```

**Simpson's:**

```
17861.5
9054.43
8041.98
1136.85
-1427.25
848.195
```

**Exact value (note: we can put //N at the end to get numerical value!)**

```
Out[8]= 1013.17
```

Results from While loop (again, you can also keep updating n manually without using a While loop)

Note: Now we want +-0.01 so the desire approximation should be between **exact\*1.01** and **exact\*0.99** but not 1.1 and 0.9!

**Trapezoid method**

```
{96, 1023.18}
```

**Simpson's method**

```
{38, 1003.77}
```

## Q2

**Suggestion:**

You can put all four equations---exact value, midpoint, trapezoid, Simpson's in one cell.

Keep testing  $f[x\_]=1$ ,  $f[x\_]=x$ ,  $f[x\_]=x^2$ ,...and so on (copy-paste so that I can see all values), until all four values are different.

Of course, you can also test the methods one-by-one; it's up to you!

The following is an example for  $f[x\_]=1$ , the first test ( $x^0=1$  so we're testing the case  $p=0$ ). Feel free to use/modify it.

```
In[®]:= f[x_] = 1
a = 0;
b = 1;
n = 10;
width = (b - a) / n;
exact = Integrate[f[x], {x, a, b}] // N
mid = width * Sum[f[a + (i + 1/2) * width], {i, 0, n - 1}] // N
trap = width * Sum[(f[a + i * width] + f[a + (i + 1) * width]) / 2, {i, 0, n - 1}] // N
simp = (width / 3) * Sum[f[a + 2 i * width] +
4 f[a + (2 i + 1) * width] + f[a + (2 i + 2) * width], {i, 0, n/2 - 1}] // N

Out[®]= 1
Out[®]= 1.
Out[®]= 1.
Out[®]= 1.
Out[®]= 1.
Out[®]= 1.
```

**Results for other functions (higher p) (+essay questions, use your own words)**

```
Out[®]= x
Out[®]= 0.5
Out[®]= 0.5
Out[®]= 0.5
Out[®]= 0.5
Out[®]= x2
Out[®]= 0.333333
Out[®]= 0.3325
Out[®]= 0.335
Out[®]= 0.333333
```

Out[ $\circ$ ]=  $x^3$

Out[ $\circ$ ]= 0.25

Out[ $\circ$ ]= 0.24875

Out[ $\circ$ ]= 0.2525

Out[ $\circ$ ]= 0.25

Out[ $\circ$ ]=  $x^4$

Out[ $\circ$ ]= 0.2

Out[ $\circ$ ]= 0.198336

Out[ $\circ$ ]= 0.20333

Out[ $\circ$ ]= 0.200013