10/15/2019

MATH241-073D/101D Discussion Worksheet
Differentiation Rules from Sec.3.1–Sec.3.6

Sec 3.1

- **The Power Rule** For any real number "n",

$$(x^n)' = \_\_\_\_\_\_\_\_$$

- $(c)' =$____, $(e^x)' =$____.
- $(f+g)' =$________, $(f-g)' =$________, $(cf)' =$________.

Sec.3.2

- **The Product Rule**

$$(fg)' = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- **The Quotient Rule**

$$\left(\frac{f}{g}\right)' = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Sec 3.3

- **Derivatives of trigonometric functions** Must remember $(\sin x)'$ and $(\cos x)'$. For the other four, use Quotient Rule or Chain Rule, or remember by "tss sst" trick.

$(\sin x)' =$ ___________________ $(\cos x)' =$ ___________________

$(\tan x)' =$ ___________________ $(\sec x)' =$ ___________________

$(\cot x)' =$ ___________________ $(\csc x)' =$ ___________________

Sec 3.4

- **The Chain Rule** (do examples!)
- $(b^x)' =$____________

Sec 3.5

- **Implicit differentiation** (do examples!)
Given an implicit equation (a curve), say $x^2 + y^2 = 25$. Find y' or dy/dx:

  1. Take $\frac{d}{dx}$ on both sides:

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}25$$

2. Apply differentiation rules term-by-term. Note that y is a function of x, so things with y needs Chain Rule:
   $2x + 2yy' = 0$
3. Solve y' in terms of x, y:
   $2yy' = -2x \Rightarrow y' = -x/y$.

If in addition, we want y' at a particular point (e.g., the slope of the tangent line at the point $(3, 4)$)

4. Plug in the given point (x, y) into y' from Step 3:
$$y'\big|_{(x,y)=(3,4)} = -3/4.$$

- **Derivatives of inverse trigonometric functions**
  Note: $\arcsin x = \sin^{-1} x \neq \frac{1}{\sin x}$.

  $(\arcsin x)' =$ ________________ $(\arctan x)' =$ ________________ $(\operatorname{arcsec} x)' =$ ________________

  $(\arccos x)' =$ ________________ $(\operatorname{arccot} x)' =$ ________________ $(\operatorname{arccsc} x)' =$ ________________

Sec 3.6

- $(\ln x)' =$ __________
- $(\log_b x)' =$ __________
- **Four cases for exponents and bases**

Cases	Differentiation rules	Examples
Constant base, constant exponent	$(b^n)' = 0$	e^π, π^e, $e^{\ln 2}$, $(\ln 2)^{\ln 3}$
Variable base, constant exponent	$\left(\left(f(x)\right)^n\right)' = n\left(f(x)\right)^{n-1} \cdot f'(x)$	$(x^2+1)^5$, $(\sin x + e^x)^{\ln 5}$
Constant base, variable exponent	$\left(b^{g(x)}\right)' = b^{g(x)} \ln b \cdot g'(x)$	2^{t^3}, $e^{\sin x}$
Variable base, variable exponent	logarithmic differentiation, or use $y = e^{\ln y}$ and use Chain Rule	x^x, $(\ln x)^{\sin x}$, $x^{\ln x}$

- **Logarithmic differentiation** (not in LQ2) (do examples!)
  Two examples: Given $f(x)^{g(x)}$, say $y = x^x$, or a complicate expression, say $y = \frac{(2x+1)^3 x^2}{(x^2+2)^4}$. Find y'.

  1. Take ln on both sides and simplify:
     $\ln y = \ln x^x = x \ln x$
     $\ln y = \ln\left(\frac{(2x+1)^3 x^2}{(x^2+2)^4}\right) = 3\ln(2x+1) + 2\ln x - 4\ln(x^2+2)$
  2. Implicit differentiation:
     $\frac{y'}{y} = (x)' \ln x + x(\ln x)' = \ln x + x\frac{1}{x} = \ln x + 1$
     $\frac{y'}{y} = \frac{3}{2x+1} \cdot (2x+1)' + \frac{2}{x} - \frac{4}{x^2+2} \cdot (x^2+2)' = \frac{3}{2x+1} \cdot 2 + \frac{2}{x} - \frac{4}{x^2+2} \cdot 2x = \frac{6}{2x+1} + \frac{2}{x} - \frac{8x}{x^2+2}$
  3. Solve y' and write y back to in terms of x.
     $y' = y(\ln x + 1) = x^x(\ln x + 1)$
     $y' = y\left(\frac{6}{2x+1} + \frac{2}{x} - \frac{8x}{x^2+2}\right) = \frac{(2x+1)^3 x^2}{(x^2+2)^4}\left(\frac{6}{2x+1} + \frac{2}{x} - \frac{8x}{x^2+2}\right)$