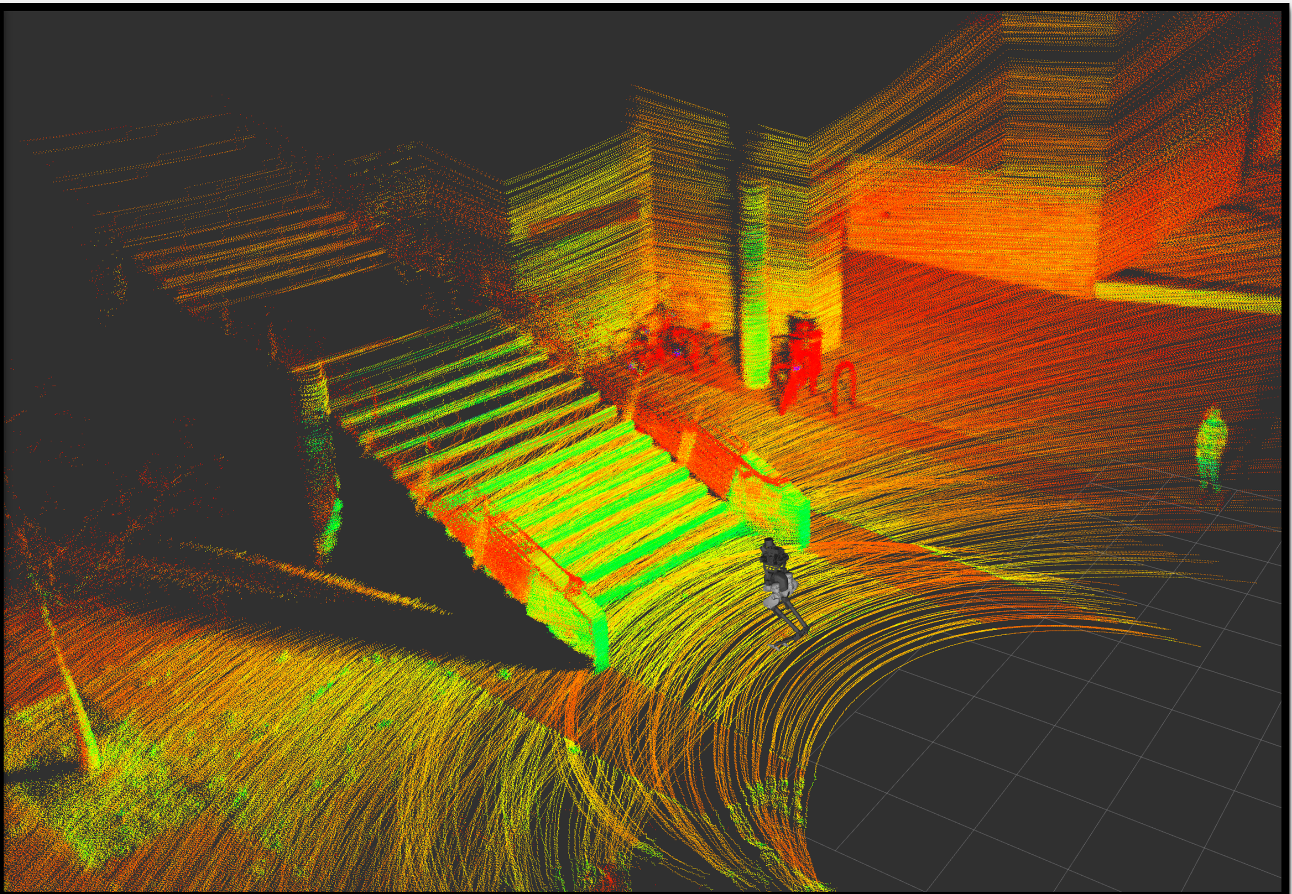
Contact-aided Invariant Extended Kalman Filtering for Legged Robot State Estimation

Ross Hartley



M

$\dot{x} = f(x,u)$

BIPED ROBOTICS LAB@MICHIGAN

Why do we need legged robots?

All terrain access

Inspection

Delivery and home robots

Search and rescue


Pavement
ROBOTICS

What states need to be estimated?

- position
- orientation
- velocity
- joint positions/velocities
- contact states

What states need to be estimated?

- position
- orientation
- velocity
- ~~joint positions/velocities~~
- ~~contact states~~

Visual-inertial odometry?

encoders

contact sensors

Failures of visual-inertial odometry

Vision may fail when …

- **Scarcity of features**
  - ➢ snow, grass …
- **Poor lighting**
  - ➢ sun glare, night …
- **Obstructions**
  - ➢ smoke, water on lens, …
- **Dynamic environments**
- **Motion blur**

Failures of visual-inertial odometry

Vision failure

Visual-inertial odometry → Inertial odometry

Integration of IMU measurements alone leads to significant drift due to sensor noise and bias!

VectorNav-100 IMU

Grade	Accelerometer Bias Error [mg]	Horizontal Position Error [m] 1s	10s	60s	1hr
Navigation	0.025	0.13 mm	12 mm	0.44 m	1.6 km
Tactical	0.3	1.5 mm	150 mm	5.3 m	19 km
Industrial	3	15 mm	1.5 m	53 m	190 km
Automotive	125	620 mm	60 m	2.2 km	7900 km

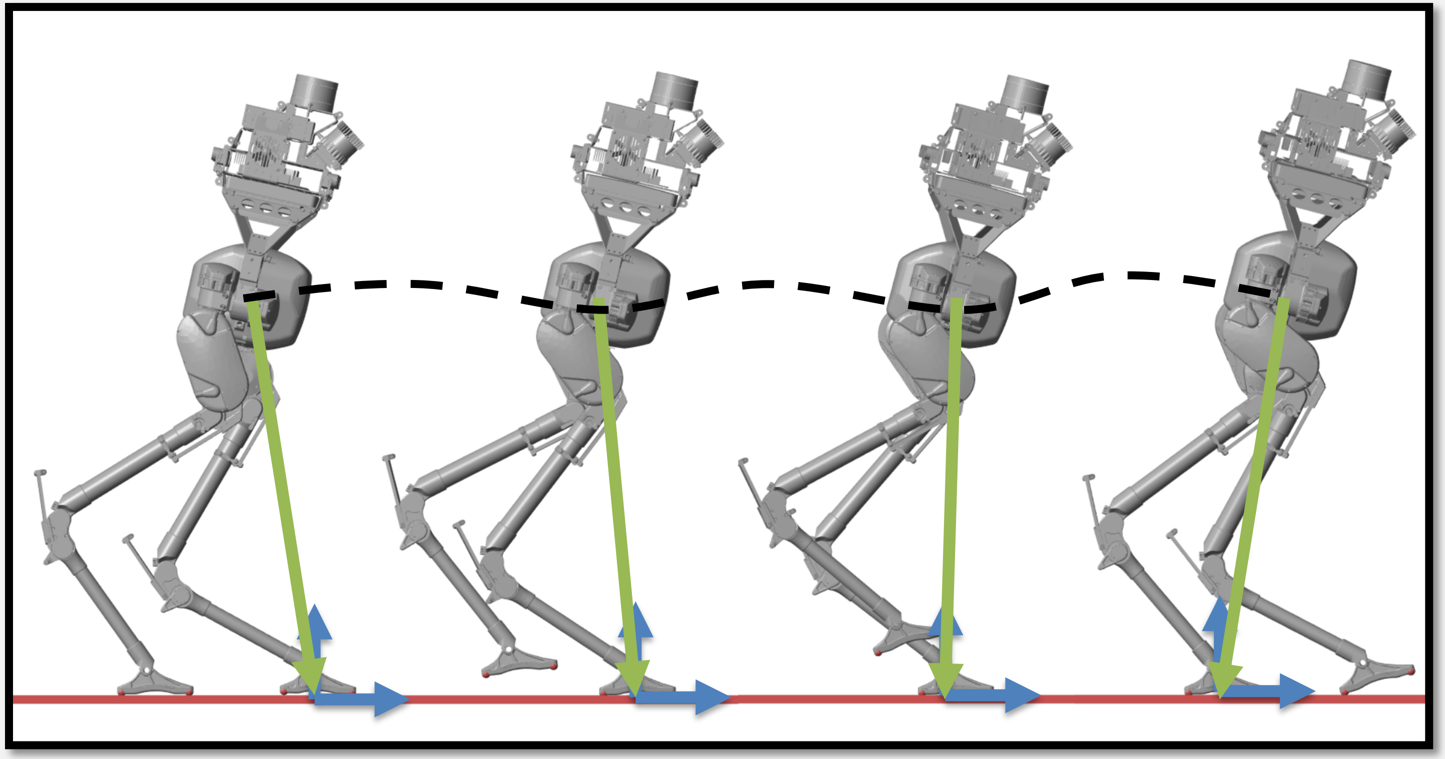
https://www.vectornav.com/support/library/imu-and-ins

Kinematic Odometry!

Odometry from **joint encoders** and **contact sensors**

- Contact *implies* that the stance foot remains fixed
- Forward kinematics is used to measure the foot position relative to the IMU
- Together these measurements can be used to estimate relative movement
- EKF to fuse with inertial data

[Bloesch 2008]



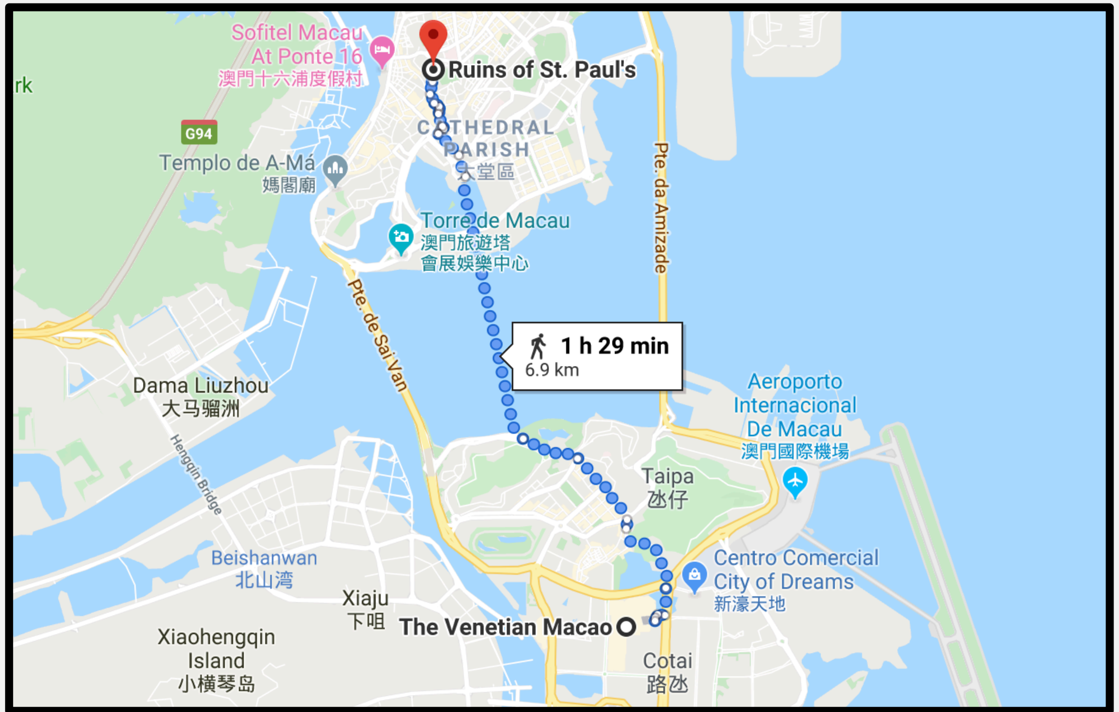
Dual-estimator approach

We want stability and autonomy!

- Only local map information is needed
- High-frequency orientation/velocity for control

- Global map and pose needed for long-term planning
- Low-frequency estimate of pose is fine

Invariant-EKF fusing **inertial, encoder, and contact measurements** (proprioceptive only)



Factor graph smoother that fuses all measurements including **vision-based loop closures (SLAM)**

Types of Kalman Filters

- **Kalman Filter:** optimal linear filter – state assumed to be a Gaussian random variable

$$\frac{\mathrm{d}}{\mathrm{dt}}\mathbf{x}_t = \mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t + \mathbf{w}_t$$

- **(Direct) Extended Kalman Filter (EKF):** linearize nonlinear **state** dynamics (filter states directly)

$$\frac{\mathrm{d}}{\mathrm{dt}}\mathbf{x}_t = f(\mathbf{x}_t, \mathbf{u}_t, \mathbf{w}_t) \qquad \mathbf{A}_t = \left.\frac{\partial f}{\partial \mathbf{x}_t}\right|_{\mathbf{x}=\bar{\mathbf{x}}_t}$$

- **(Indirect or Error-State) EKF:** linearize nonlinear **error** dynamics (filter error states)

$$\mathbf{e}_t \triangleq \mathbf{x}_t \boxminus \hat{\mathbf{x}}_t \qquad \begin{aligned}\frac{\mathrm{d}}{\mathrm{dt}}\mathbf{e}_t &= g(\mathbf{e}_t, \mathbf{x}_t, \mathbf{u}_t, \mathbf{w}_t) \\ &\approx \mathbf{A}_t(\bar{\mathbf{x}}_t, \mathbf{u}_t)\, \mathbf{e}_t + \bar{\mathbf{w}}_t\end{aligned}$$

In general, the linearization depends on the current state estimate.

Bad estimate → **Incorrect linearization** → **Poor performance**

We can choose the error-state!

- **(Indirect or Error-State) EKF:** linearize nonlinear **error** dynamics (filter error states)

$$\mathbf{e}_t \triangleq \mathbf{x}_t \boxminus \hat{\mathbf{x}}_t \qquad \frac{\mathrm{d}}{\mathrm{dt}}\mathbf{e}_t = g(\mathbf{e}_t, \mathbf{x}_t, \mathbf{u}_t, \mathbf{w}_t)$$

$$\approx \mathbf{A}_t(\bar{\mathbf{x}}_t, \mathbf{u}_t)\,\mathbf{e}_t + \bar{\mathbf{w}}_t$$

Different choices of error variables lead to different linearized error dynamics

e.g. Euler angle error vs. quaternion error

Is there a choice of error variables that leads to autonomous dynamics?

(independent of the state estimate)

Yes! (group-affine systems) → Invariant Extended Kalman Filter

[Barrau 2014]

Lie Group Theory – Crash Course

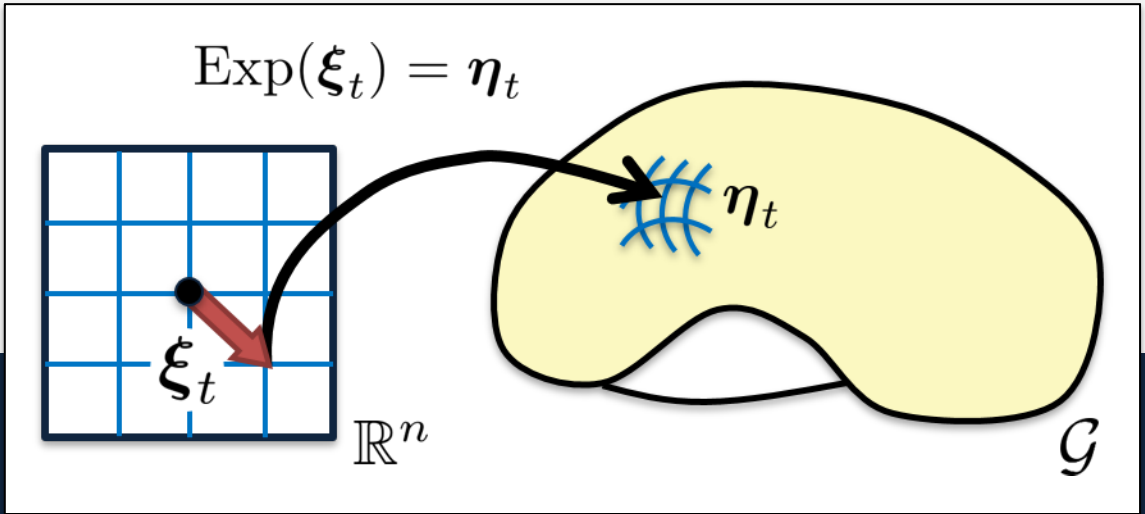
- A **Lie group**, $\mathcal{G}$, is a group that is also a differentiable manifold.
  - Examples: SO(3), SE(3) are matrix Lie groups

- The **Lie algebra** is defined as the tangent space at the identity element of the group $\mathcal{T}_e\mathcal{G}$
  - This vector space is isomorphic to $\mathbb{R}^n$

- The group's **exponential map**, $\exp : \mathcal{T}_e\mathcal{G} \rightarrow \mathcal{G}$, maps a vector in the Lie algebra to the Lie group
  - Inverse is the **logarithm map**, $\log : \mathcal{G} \rightarrow \mathcal{T}_e\mathcal{G}$

"hat operator"
$$(\cdot)^\wedge : \mathbb{R}^n \rightarrow \mathcal{T}_e\mathcal{G}$$

Vectorized notation:
$$\mathrm{Exp}(\boldsymbol{\xi}) \triangleq \exp\left(\boldsymbol{\xi}^\wedge\right)$$
$$\mathrm{Log}\left(\mathrm{Exp}(\boldsymbol{\xi})\right) = \boldsymbol{\xi}$$



Invariant Kalman Filtering

- System defined on a **matrix Lie group**: $\mathbf{X}_t \in \mathcal{G}$ $\quad \mathrm{SO}(3), \mathrm{SE}(3), \mathrm{etc.}$

- Dynamics satisfy "**group affine**" property: $f_{u_t}(\mathbf{X}_1\mathbf{X}_2) = f_{u_t}(\mathbf{X}_1)\mathbf{X}_2 + \mathbf{X}_1 f_{u_t}(\mathbf{X}_2) - \mathbf{X}_1 f_{u_t}(\mathbf{I}_d)\mathbf{X}_2$

With "group affine" systems, a particular choice of error variables will lead to log-linear error dynamics.

- Error is defined through matrix multiplication:

$$\boldsymbol{\eta}_t^r = \bar{\mathbf{X}}_t \mathbf{X}_t^{-1} \quad \text{(Right-Invariant Error)}$$

$$\boldsymbol{\eta}_t^l = \mathbf{X}_t^{-1} \bar{\mathbf{X}}_t \quad \text{(Left-Invariant Error)}$$

True State

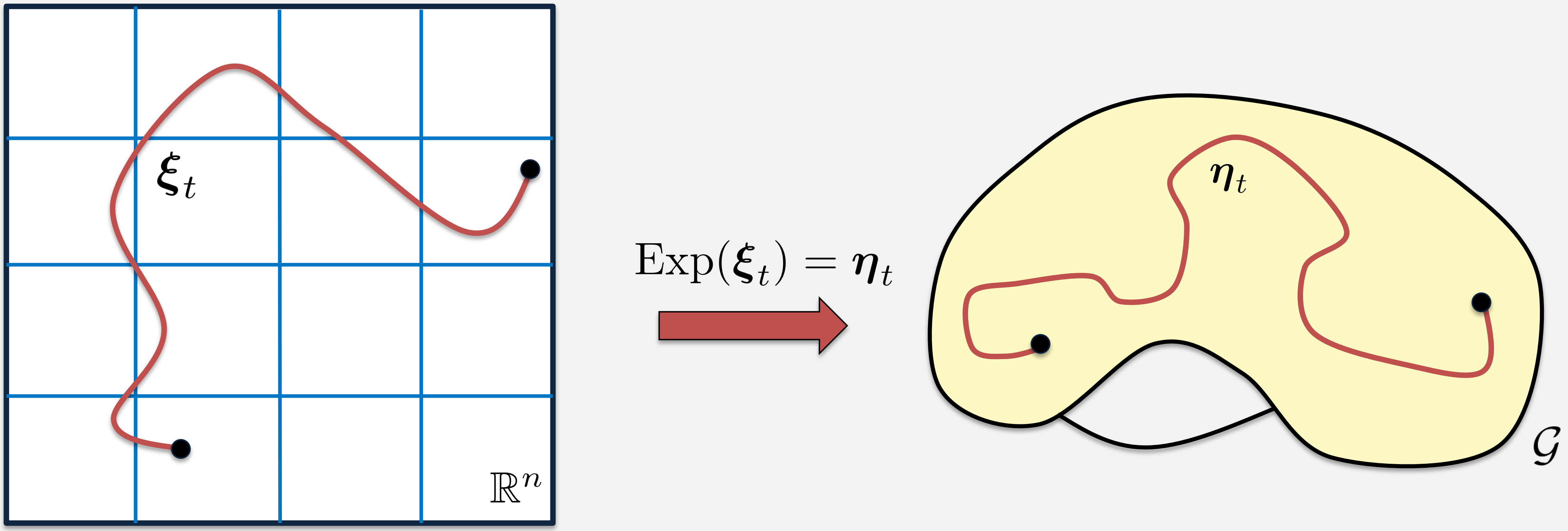
Estimate

Error is invariant to (right or left) actions of the group

$$(\mathbf{L}\mathbf{X}_t)^{-1}\mathbf{L}\bar{\mathbf{X}}_t = \mathbf{X}_t^{-1}\mathbf{L}^{-1}\mathbf{L}\bar{\mathbf{X}}_t = \mathbf{X}_t^{-1}\bar{\mathbf{X}}_t$$

[Barrau and Bonnabel 2017]

Log-Linear Error Dynamics



$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\xi}_t = \mathbf{A}_t \boldsymbol{\xi}_t$$

The nonlinear error dynamics is **exactly** determined by a linear system in the Lie algebra!!!

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\eta}_t = g_{u_t}(\boldsymbol{\eta}_t)$$

[Barrau and Bonnabel 2017]

Invariant Observations

$$\mathbf{Y}_t = \mathbf{X}_t^{-1}\mathbf{b} + \mathbf{V}_t \quad \text{(Right Invariant Observation)}$$

$$\mathbf{Y}_t = \mathbf{X}_t\mathbf{b} + \mathbf{V}_t \quad \text{(Left Invariant Observation)}$$

Autonomous innovation equations!

measurement = state * constant + noise

Both the linearized dynamics and measurement models will be state independent!

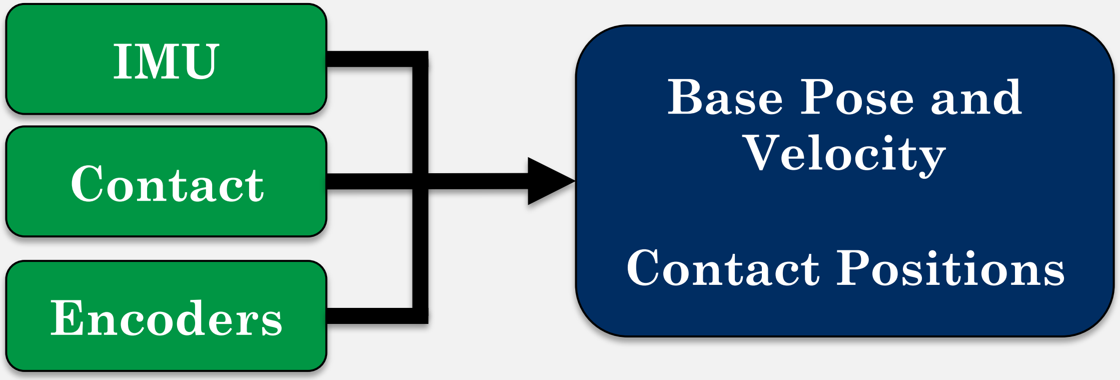
Contact-Aided Invariant EKF

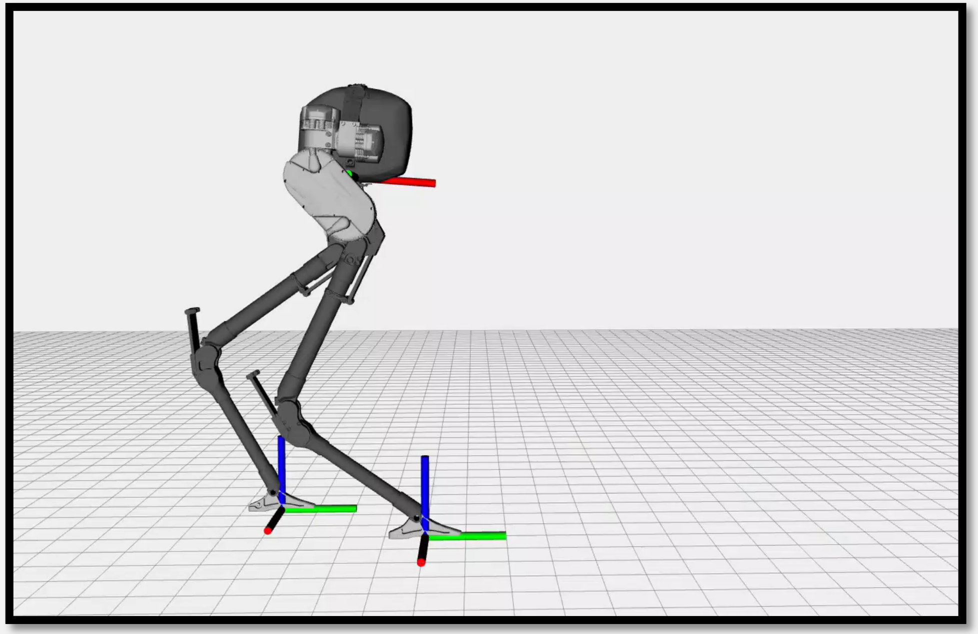
Propagation

- Use **IMU** measurements to predict base frame movement.
- Use **contact** sensor measurement to predict supporting feet movement (zero translation).

Correction

- Use encoder measurements and **forward kinematics** to correct state estimate.





[Hartley et al. RSS 2018]
[Hartley et al. IJRR 2019]

States and Inputs

- The state is expressed as a matrix Lie group, $\mathbf{X}_t \in \mathrm{SE}_K(3)$

$$\mathbf{X}_t \triangleq \begin{bmatrix} \overbrace{\mathbf{R}_{\mathrm{WB}}(t) & {}_{\mathrm{W}}\mathbf{v}_{\mathrm{WB}}(t) & {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WB}}(t)}^{\text{Orientation, velocity, and position of IMU in world frame}} & \overbrace{{}_{\mathrm{W}}\mathbf{p}_{\mathrm{WC}_1}(t) & \cdots & {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WC}_N}(t)}^{\text{Position of contact points in world frame}} \\ \mathbf{0}_{1,3} & 1 & 0 & 0 & \cdots & 0 \\ \mathbf{0}_{1,3} & 0 & 1 & 0 & \cdots & 0 \\ \mathbf{0}_{1,3} & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{1,3} & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \triangleq \underbrace{\begin{bmatrix} \mathbf{R}_t & \mathbf{v}_t & \mathbf{p}_t & \mathbf{d}_t \\ \mathbf{0}_{1,3} & 1 & 0 & 0 \\ \mathbf{0}_{1,3} & 0 & 1 & 0 \\ \mathbf{0}_{1,3} & 0 & 0 & 1 \end{bmatrix}}_{\text{Shorthand notation (only one contact)}}$$

$$\mathbf{u}_t \triangleq \begin{bmatrix} {}_{\mathrm{B}}\tilde{\boldsymbol{\omega}}_{\mathrm{WB}}(t) \\ {}_{\mathrm{B}}\tilde{\mathbf{a}}_{\mathrm{WB}}(t) \end{bmatrix} \triangleq \begin{bmatrix} \tilde{\boldsymbol{\omega}}_{\mathrm{t}} \\ \tilde{\mathbf{a}}_{\mathrm{t}} \end{bmatrix}$$

accelerometer and gyroscope measurements

[Hartley et al. RSS 2018]
[Hartley et al. IJRR 2019]

Inertial-Contact Dynamics Model

$$\cancel{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu + J^T(q)F}$$

Do we have to use to robot's complicated dynamics?

No!

The "strapdown" inertial-contact model circumvents using the full dynamics

$$\begin{aligned}
\dot{\mathbf{R}}_t &= \mathbf{R}_t \left(\tilde{\boldsymbol{\omega}}_t - \mathbf{w}_t^g\right)_\times \\
\dot{\mathbf{v}}_t &= \mathbf{R}_t \left(\tilde{\mathbf{a}}_t - \mathbf{w}_t^a\right) + \mathbf{g} \\
\dot{\mathbf{p}}_t &= \mathbf{v}_t \\
\dot{\mathbf{d}}_t &= \mathbf{R}_t \, \mathbf{R}_{\mathrm{BC}}(\tilde{\boldsymbol{\alpha}}_t) \left(-\mathbf{w}_t^v\right)
\end{aligned}$$

orientation of contact frame
with respect to body frame

Inertial-Contact Dynamics Model

$$\cancel{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu + J^T(q)F}$$

The "strapdown" inertial-contact model circumvents using the full dynamics

$$\begin{aligned}
\dot{\mathbf{R}}_t &= \mathbf{R}_t \left(\tilde{\boldsymbol{\omega}}_t - \mathbf{w}_t^g\right)_\times \\
\dot{\mathbf{v}}_t &= \mathbf{R}_t \left(\tilde{\mathbf{a}}_t - \mathbf{w}_t^a\right) + \mathbf{g} \\
\dot{\mathbf{p}}_t &= \mathbf{v}_t \\
\dot{\mathbf{d}}_t &= \mathbf{R}_t \, \mathbf{R}_{\mathrm{BC}}(\tilde{\boldsymbol{\alpha}}_t) \left(-\mathbf{w}_t^v\right)
\end{aligned}$$

orientation of contact frame
with respect to body frame

Written in matrix form:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{X}_t &= \begin{bmatrix} \mathbf{R}_t\left(\tilde{\boldsymbol{\omega}}_t\right)_\times & \mathbf{R}_t\tilde{\mathbf{a}}_t + \mathbf{g} & \mathbf{v}_t & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & 0 & 0 & 0 \\ \mathbf{0}_{1\times3} & 0 & 0 & 0 \\ \mathbf{0}_{1\times3} & 0 & 0 & 0 \end{bmatrix} \\
&\quad - \begin{bmatrix} \mathbf{R}_t & \mathbf{v}_t & \mathbf{p}_t & \mathbf{d}_t \\ \mathbf{0}_{1\times3} & 1 & 0 & 0 \\ \mathbf{0}_{1\times3} & 0 & 1 & 0 \\ \mathbf{0}_{1\times3} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \left(\mathbf{w}_t^g\right)_\times & \mathbf{w}_t^a & \mathbf{0}_{3\times1} & \mathbf{R}_{\mathrm{BC}}(\tilde{\boldsymbol{\alpha}}_t)\mathbf{w}_t^v \\ \mathbf{0}_{1\times3} & 0 & 0 & 0 \\ \mathbf{0}_{1\times3} & 0 & 0 & 0 \\ \mathbf{0}_{1\times3} & 0 & 0 & 0 \end{bmatrix} \\
&\triangleq f_{u_t}(\mathbf{X}_t) - \mathbf{X}_t\left(\mathbf{w}_t\right)^\wedge
\end{aligned}$$

Satisfies group affine property!

Log-Linear Error Dynamics

(Right Invariant Error)

$$\boldsymbol{\eta}_t^r \triangleq \bar{\mathbf{X}}_t \mathbf{X}_t^{-1} = \mathrm{Exp}(\boldsymbol{\xi}_t)$$

$$\mathbf{A}_t = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (\mathbf{g})_\times & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\xi}_t = \mathbf{A}_t \boldsymbol{\xi}_t + \mathrm{Ad}_{\bar{\mathbf{X}}_t} \mathbf{w}_t$$

Linearized error dynamics matrix is independent of the state estimate!

(Left Invariant Error)

$$\boldsymbol{\eta}_t^l \triangleq \mathbf{X}_t^{-1} \bar{\mathbf{X}}_t = \mathrm{Exp}(\boldsymbol{\xi}_t)$$

$$\mathbf{A}_t = \begin{bmatrix} -(\tilde{\boldsymbol{\omega}}_t)_\times & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -(\tilde{\mathbf{a}}_t)_\times & -(\tilde{\boldsymbol{\omega}}_t)_\times & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & -(\tilde{\boldsymbol{\omega}}_t)_\times & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -(\tilde{\boldsymbol{\omega}}_t)_\times \end{bmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\xi}_t = \mathbf{A}_t \boldsymbol{\xi}_t + \mathbf{w}_t$$

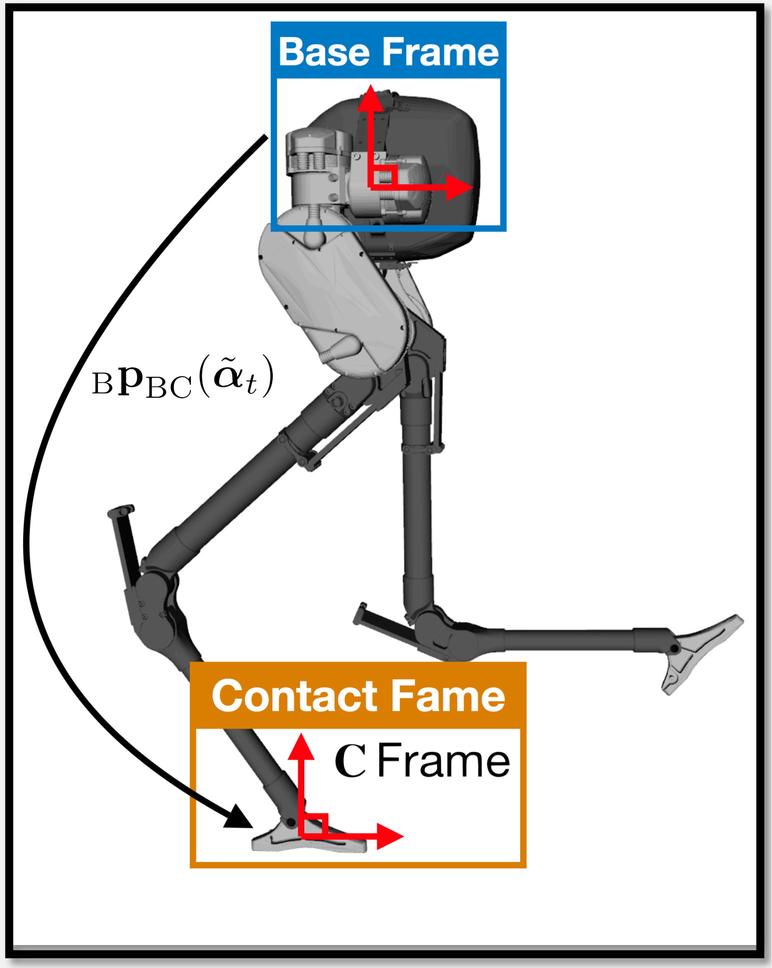
Forward Kinematic Position Measurements

Using forward kinematics, we can measure the position of the contact frame relative to the base (IMU) frame:

$$ {}_{\mathrm{B}}\mathbf{p}_{\mathrm{BC}}(\tilde{\boldsymbol{\alpha}}_t) \approx \mathbf{R}_t^{\top}(\mathbf{d}_t - \mathbf{p}_t) + {}_{\mathrm{B}}\mathbf{J}_{\mathrm{BC}}^{\dot{p}}(\tilde{\boldsymbol{\alpha}}_t)\mathbf{w}_t^{\alpha} $$

Written in matrix form:

$$ \underbrace{\begin{bmatrix} {}_{\mathrm{B}}\mathbf{p}_{\mathrm{BC}}(\tilde{\boldsymbol{\alpha}}_t) \\ 0 \\ 1 \\ -1 \end{bmatrix}}_{\mathbf{Y}_t} = \underbrace{\begin{bmatrix} \mathbf{R}_t^{\top} & -\mathbf{R}_t^{\top}\mathbf{v}_t & -\mathbf{R}_t^{\top}\mathbf{p}_t & -\mathbf{R}_t^{\top}\mathbf{d}_t \\ \mathbf{0}_{1\times3} & 1 & 0 & 0 \\ \mathbf{0}_{1\times3} & 0 & 1 & 0 \\ \mathbf{0}_{1\times3} & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{X}_t^{-1}} \underbrace{\begin{bmatrix} \mathbf{0}_{3\times1} \\ 0 \\ 1 \\ -1 \end{bmatrix}}_{\mathbf{b}} + \underbrace{\begin{bmatrix} {}_{\mathrm{B}}\mathbf{J}_{\mathrm{BC}}^{\dot{p}}(\tilde{\boldsymbol{\alpha}}_t)\mathbf{w}_t^{\alpha} \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{V}_t} $$



Has right invariant observation structure!

Linearized observation matrix is **constant**!

$$ \mathbf{H}_t = \begin{bmatrix} \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{I} \end{bmatrix} $$

Right Invariant EKF Equations

Propagation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{\mathbf{X}}_t = f_{u_t}(\bar{\mathbf{X}}_t)$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}_t = \mathbf{A}_t\mathbf{P}_t + \mathbf{P}_t\mathbf{A}_t^\mathsf{T} + \bar{\mathbf{Q}}_t,$$

Linearizations are constant!

$$\mathbf{A}_t = \begin{bmatrix} 0 & 0 & 0 & 0 \\ (\mathbf{g})_\times & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\mathbf{Q}}_t = \mathrm{Ad}_{\bar{\mathbf{X}}_t}\mathrm{Cov}(\mathbf{w}_t)\mathrm{Ad}_{\bar{\mathbf{X}}_t}^\mathsf{T}$$

Correction:

$$\bar{\mathbf{X}}_t^+ = \mathrm{Exp}\left(\overbrace{\mathbf{K}_t\mathbf{\Pi}\left(\bar{\mathbf{X}}_t\mathbf{Y}_t\right)}^{\text{correction vector}}\right)\bar{\mathbf{X}}_t$$
$$\mathbf{P}_t^+ = (\mathbf{I} - \mathbf{K}_t\mathbf{H}_t)\mathbf{P}_t$$

$$\left.\begin{aligned} \mathbf{S}_t &= \mathbf{H}_t\mathbf{P}_t\mathbf{H}_t^\mathsf{T} + \bar{\mathbf{N}}_t \\ \mathbf{K}_t &= \mathbf{P}_t\mathbf{H}_t^\mathsf{T}\mathbf{S}_t^{-1} \end{aligned}\right\} \text{Computing Kalman Gain}$$

$$\mathbf{H}_t = \begin{bmatrix} \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{I} \end{bmatrix}$$

$$\bar{\mathbf{N}}_t = \bar{\mathbf{R}}_t\, {}_\mathrm{B}\mathbf{J}_{\mathrm{BC}}^{\dot{p}}(\tilde{\boldsymbol{\alpha}}_t)\,\mathrm{Cov}(\mathbf{w}_t^\alpha)\left({}_\mathrm{B}\mathbf{J}_{\mathrm{BC}}^{\dot{p}}(\tilde{\boldsymbol{\alpha}}_t)\right)^\mathsf{T}\bar{\mathbf{R}}_t^\mathsf{T}$$

Observability Analysis

Discrete time state transition matrix:

$$\mathbf{\Phi} = \exp_m(\mathbf{A}_t \Delta t) = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ (\mathbf{g})_\times \Delta t & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \frac{1}{2}(\mathbf{g})_\times \Delta t^2 & \mathbf{I}\Delta t & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

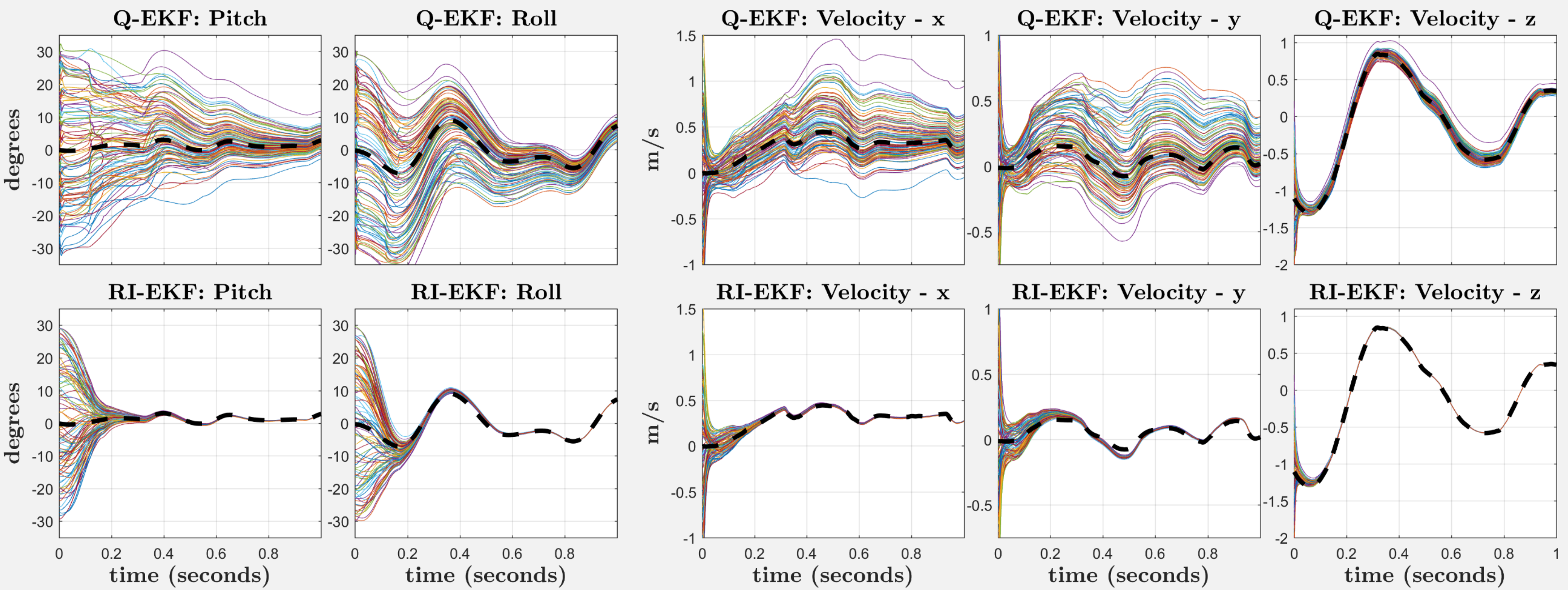
Observability matrix can be computed as:

$$\mathcal{O} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H\Phi} \\ \mathbf{H\Phi}^2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{I} \\ -\frac{1}{2}(\mathbf{g})_\times \Delta t^2 & -\mathbf{I}\Delta t & -\mathbf{I} & \mathbf{I} \\ -2(\mathbf{g})_\times \Delta t^2 & -2\mathbf{I}\Delta t^2 & -\mathbf{I} & \mathbf{I} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

- Absolute position and yaw are unobservable (drift will occur)
- Remaining states have **local stability about any trajectory**!

[Hartley et al. RSS 2018]
[Hartley et al. IJRR 2019]

Simulation Results

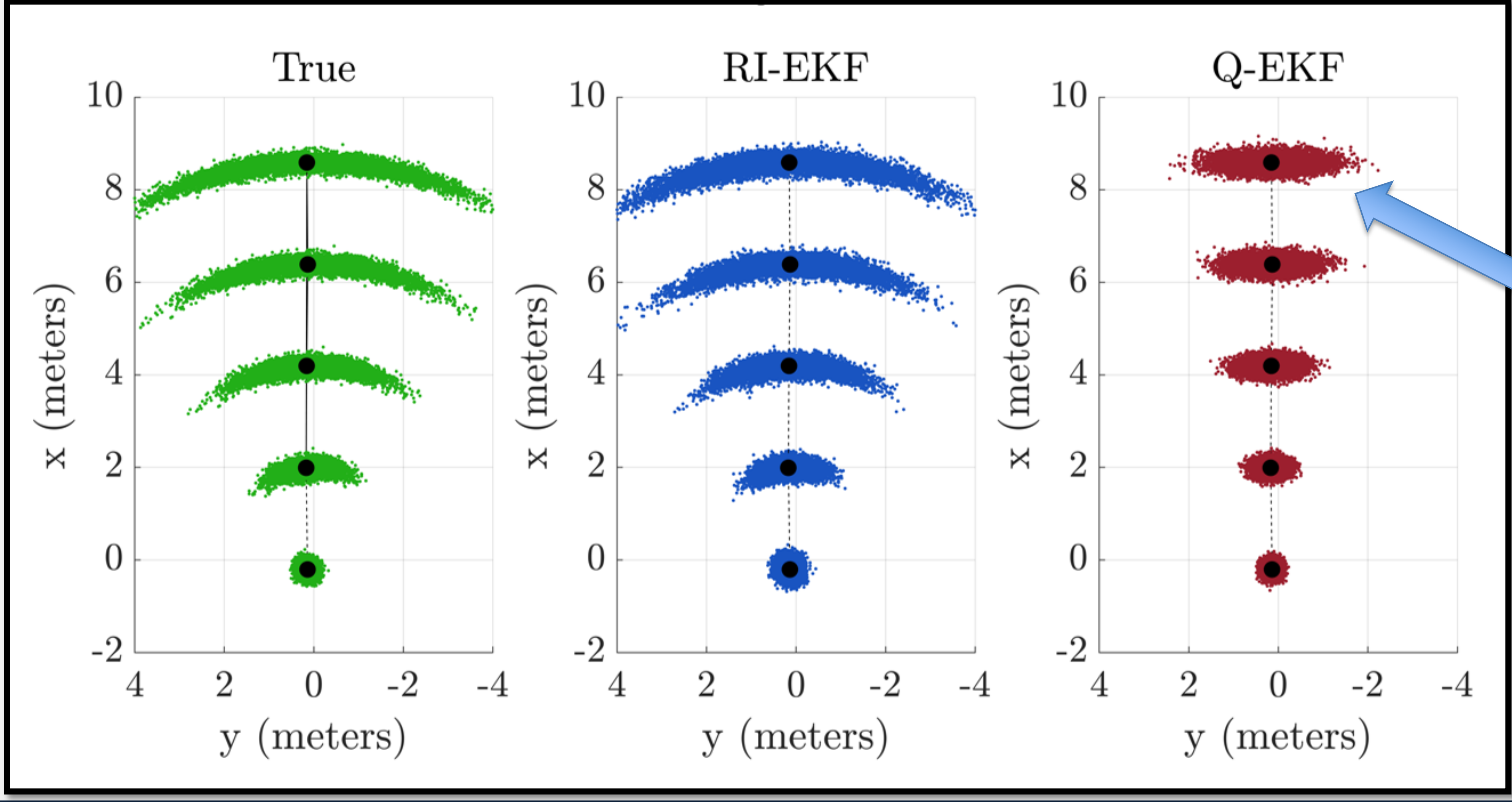


We ran 100 simulations using the same measurements and noise statistics, while randomly initializing the orientation and velocity estimates.

[Hartley et al. RSS 2018]
[Hartley et al. IJRR 2019]

Covariance Propagation

Robot walks forward with initial yaw uncertainty

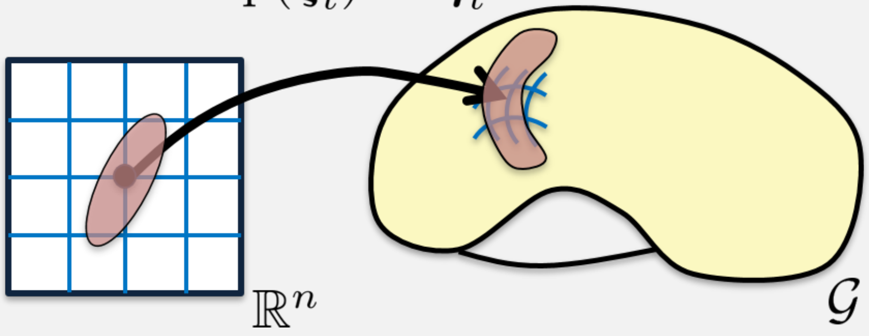


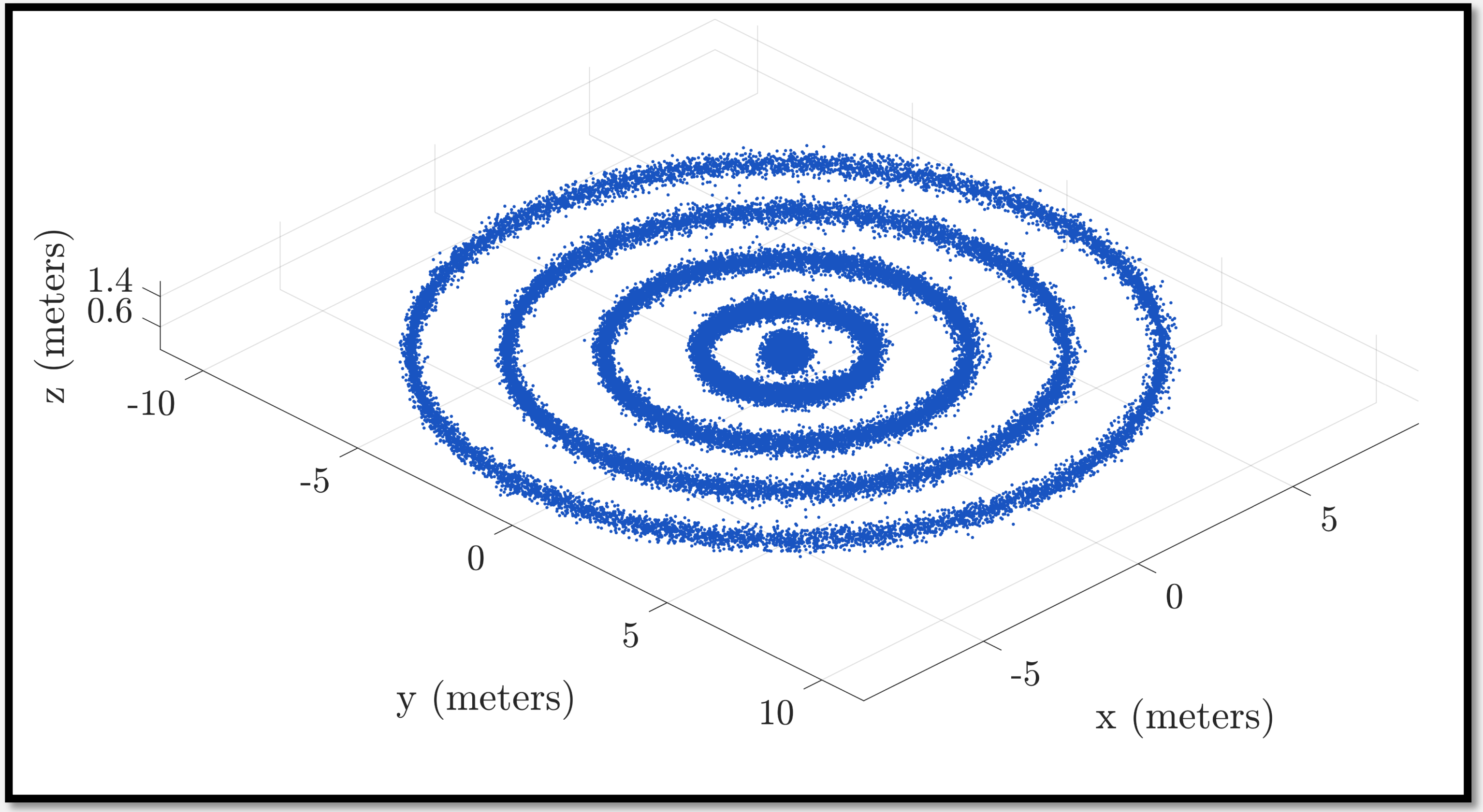
Position uncertainty cannot be captured with a simple covariance ellipse.

Walking with Unknown Initial Yaw

- Robot walking in a straight line with completely uncertain initial yaw.
- Yaw is unobservable along with absolute position

$$\mathrm{Exp}(\boldsymbol{\xi}_t) = \boldsymbol{\eta}_t$$





Incorporating IMU Bias

Unfortunately, no known way to incorporate IMU bias into the Lie group while maintaining the "group affine" property [Barrau 2015]

<u>"Imperfect" Invariant EKF</u>

State and errors become tuples:

$$(\mathbf{X}_t,\ \mathbf{b}_t) \in \mathcal{G} \times \mathbb{R}^6$$

$$\mathbf{e}_t^r \triangleq (\bar{\mathbf{X}}_t \mathbf{X}_t^{-1},\ \bar{\mathbf{b}}_t - \mathbf{b}_t)$$

Invariant EKF

Error-State EKF

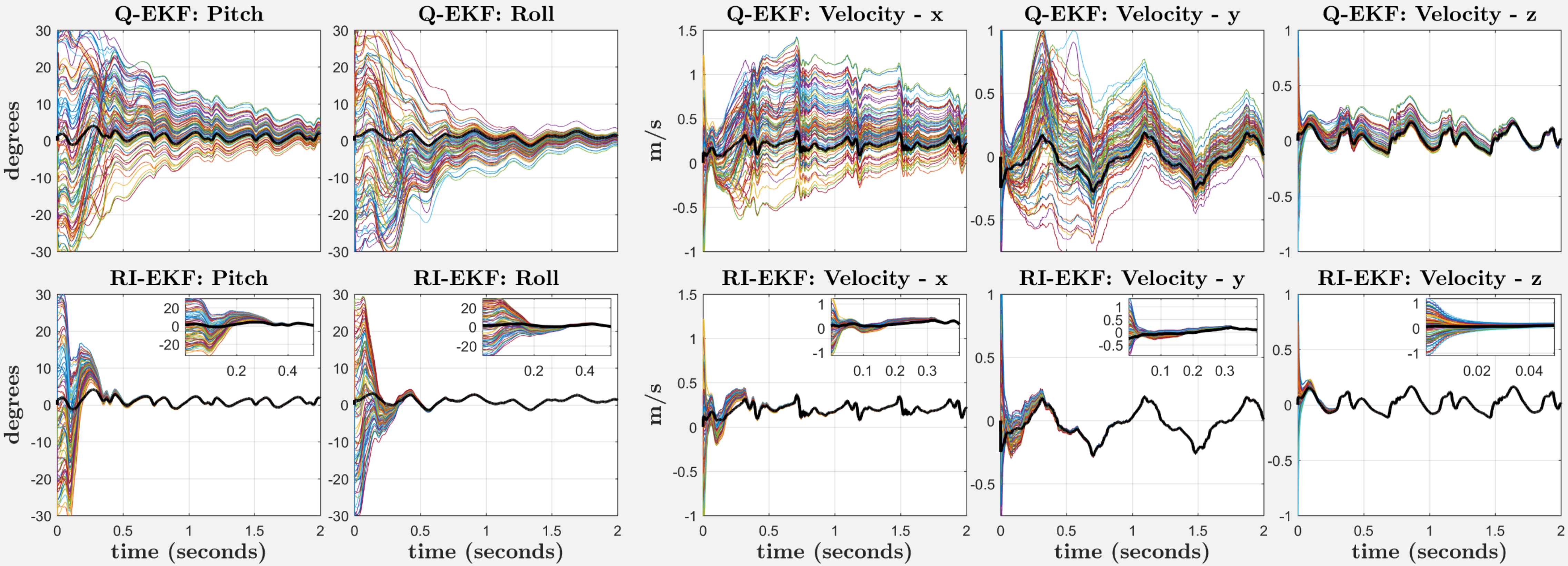
New linearized dynamics and noise matrices:

$$\mathbf{A}_t = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\bar{\mathbf{R}}_t & \mathbf{0} \\ (\mathbf{g})_\times & \mathbf{0} & \mathbf{0} & \mathbf{0} & -(\bar{\mathbf{v}})_\times \bar{\mathbf{R}}_t & -\bar{\mathbf{R}}_t \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & -(\bar{\mathbf{p}}_t)_\times \bar{\mathbf{R}}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -(\bar{\mathbf{d}}_t)_\times \bar{\mathbf{R}}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\bar{\mathbf{Q}}_t = \begin{bmatrix} \mathrm{Ad}_{\bar{\mathbf{X}}_t} & \mathbf{0}_{12,6} \\ \mathbf{0}_{6,12} & \mathbf{I}_6 \end{bmatrix} \mathrm{Cov}(\mathbf{w}_t) \begin{bmatrix} \mathrm{Ad}_{\bar{\mathbf{X}}_t} & \mathbf{0}_{12,6} \\ \mathbf{0}_{6,12} & \mathbf{I}_6 \end{bmatrix}^{\mathsf{T}}$$

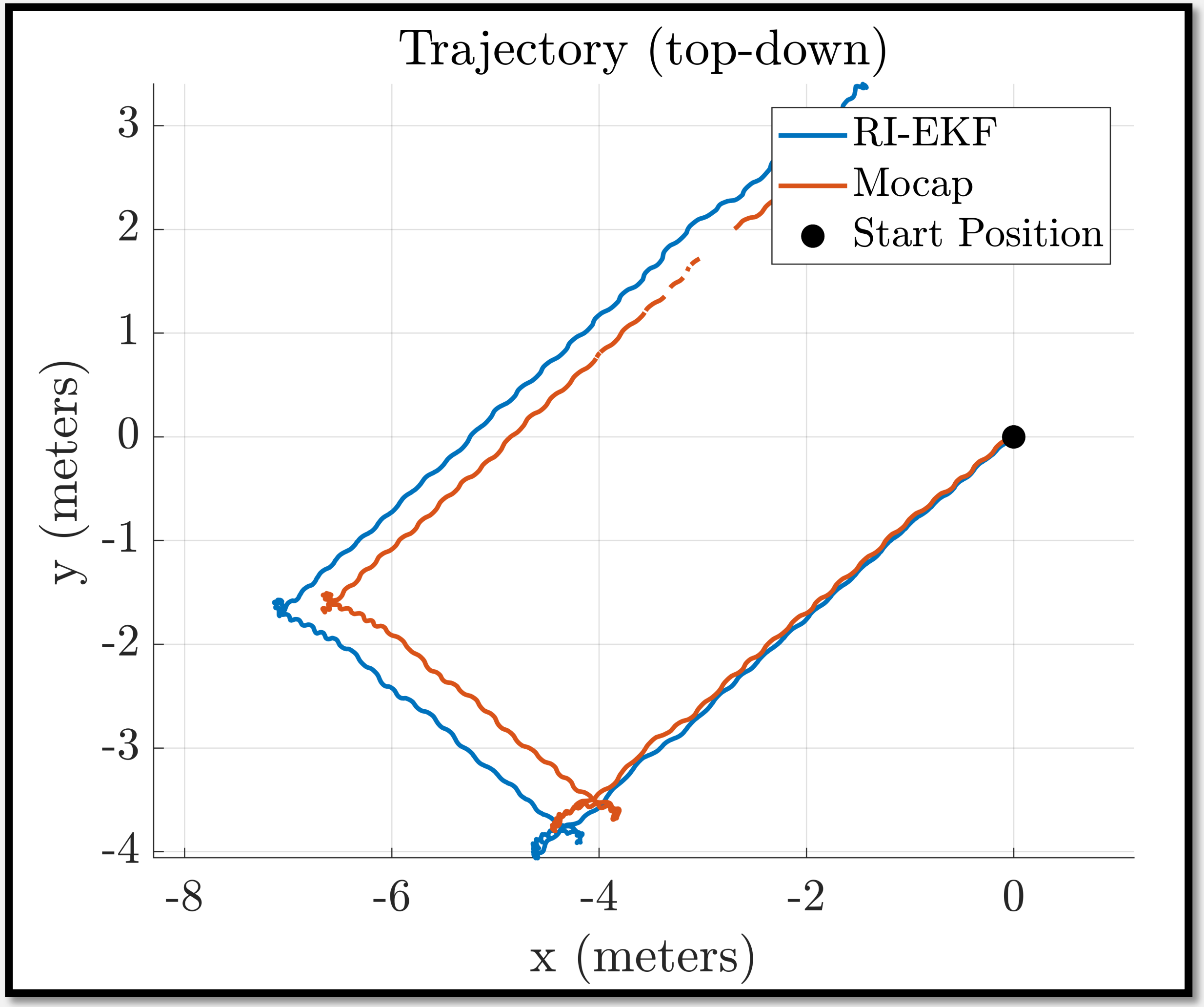
[Hartley et al. RSS 2018]
[Hartley et al. IJRR 2019]

Experimental Results



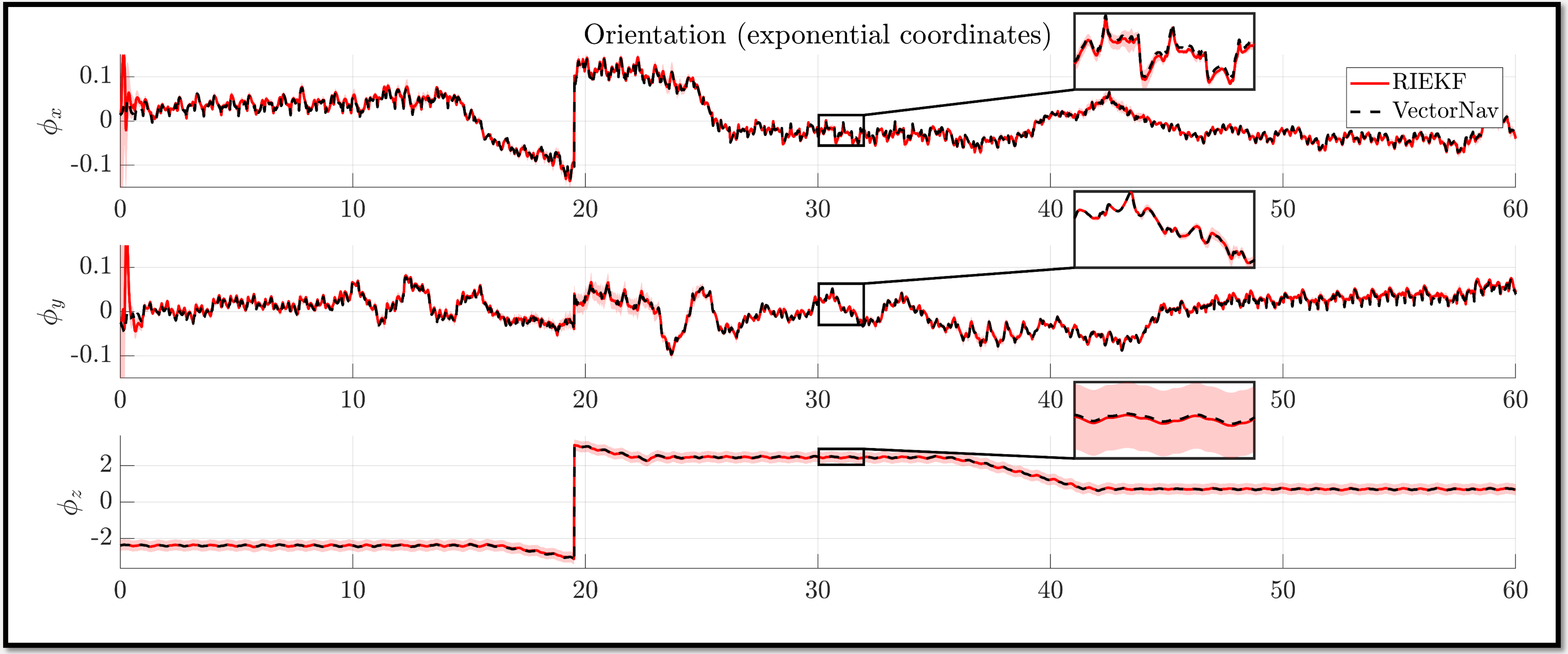
We ran the filters 100 times using the same measurements (from a walking experiment) and noise statistics, while randomly initializing the orientation and velocity estimates.

Motion Capture Experiment



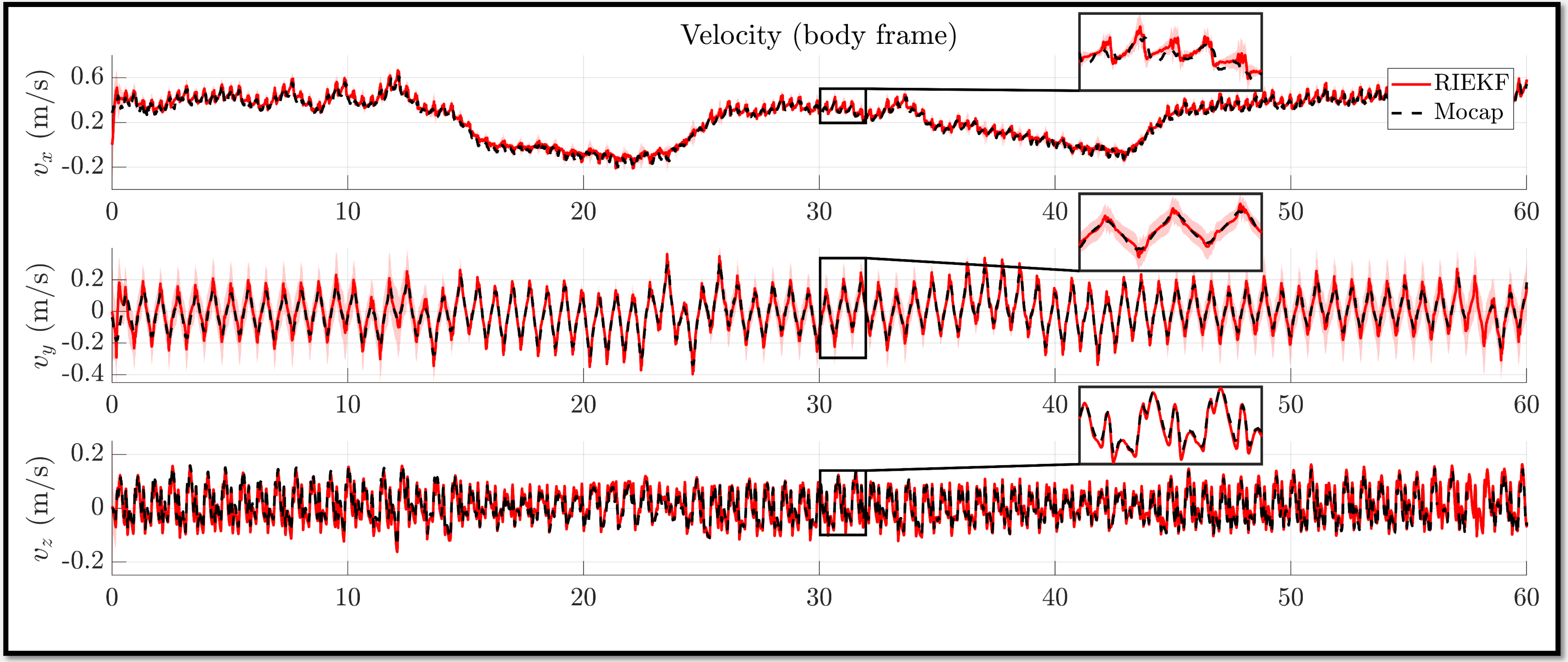
[Hartley et al. RSS 2018]
[Hartley et al. IJRR 2019]

Motion Capture Experiment



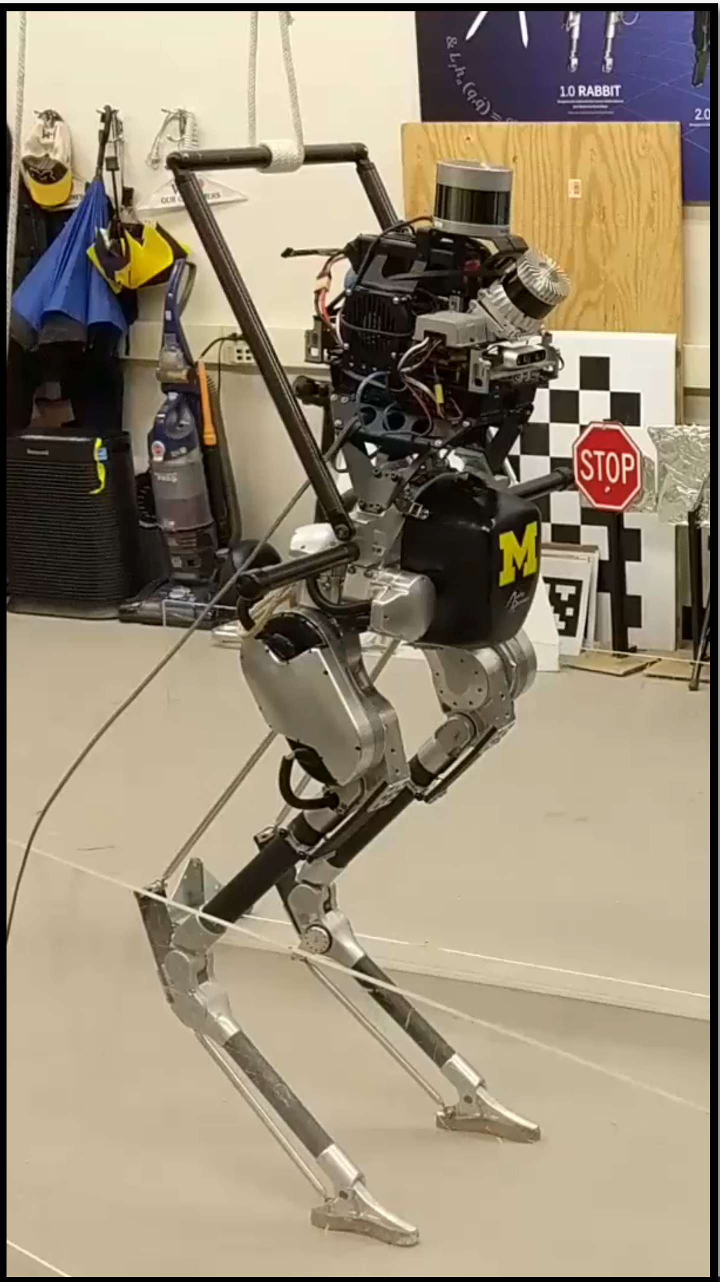
[Hartley et al. RSS 2018]
[Hartley et al. IJRR 2019]

Motion Capture Experiment



[Hartley et al. RSS 2018]
[Hartley et al. IJRR 2019]

Right Invariant EKF Trajectory
Sidewalk Followed
University of Mighigan
Wave Field
20 m



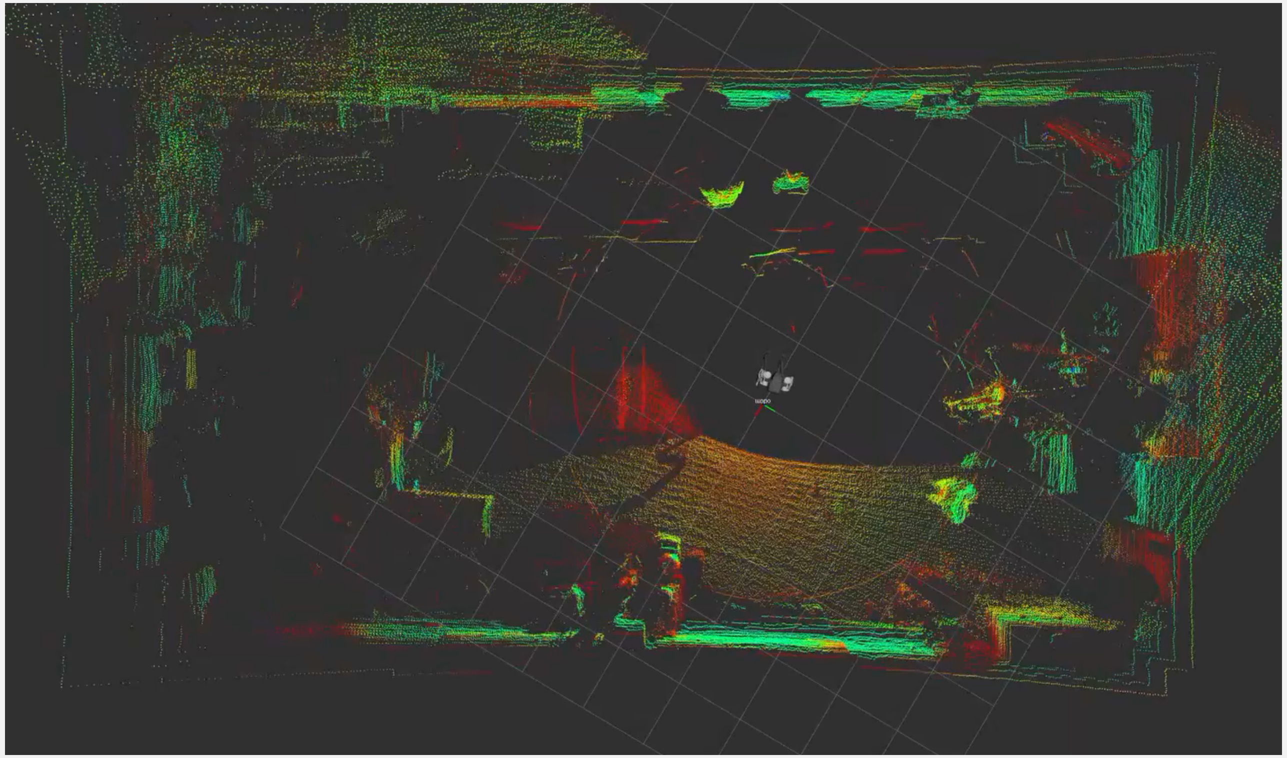
New Torso and Perception System

- Velodyne 32 beam LiDAR (10 Hz)
- Ouster 64 beam LiDAR+IMU (10 Hz)
- Two Intel RealSense depth cameras (30 Hz)
- VectorNav-100 IMU (800 Hz, in pelvis)
- Nvidia Jetson TX2 GPU
- Router, switch, power supply

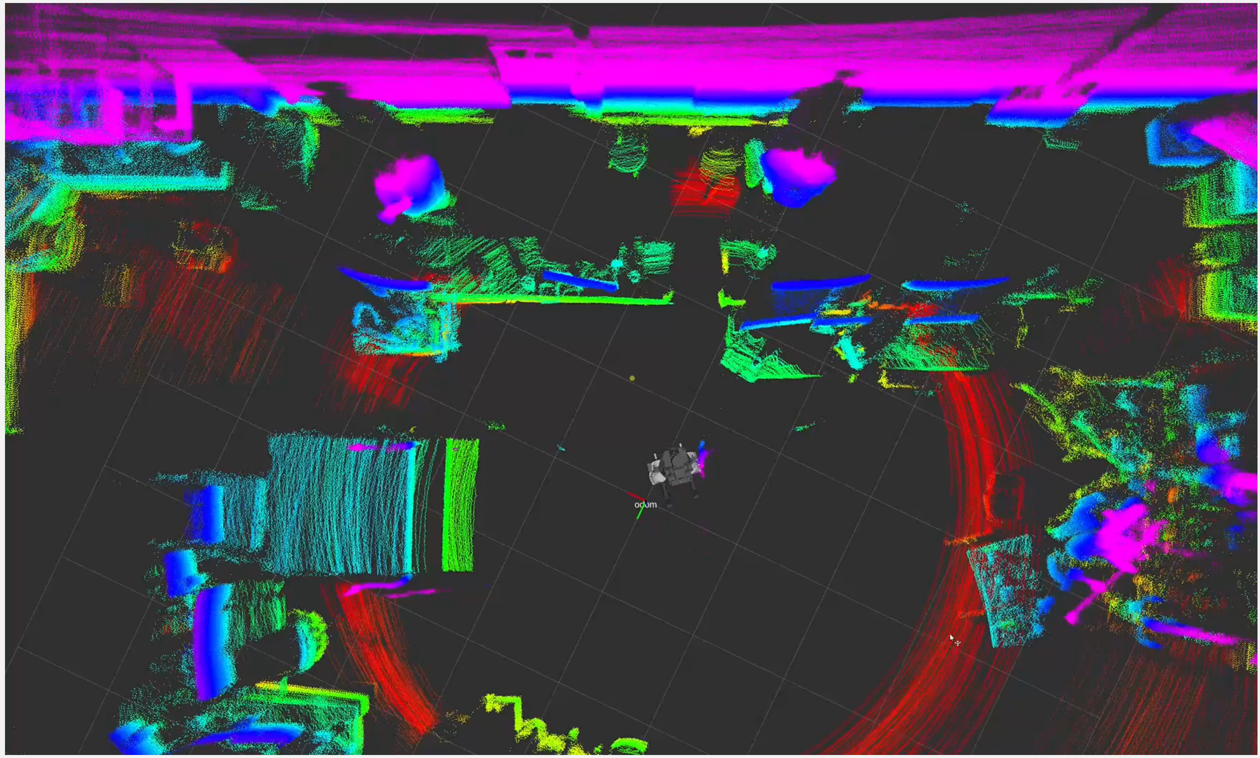
Challenges: **calibration, synchronization, data collection**

LiDAR Motion Compensation using InEKF

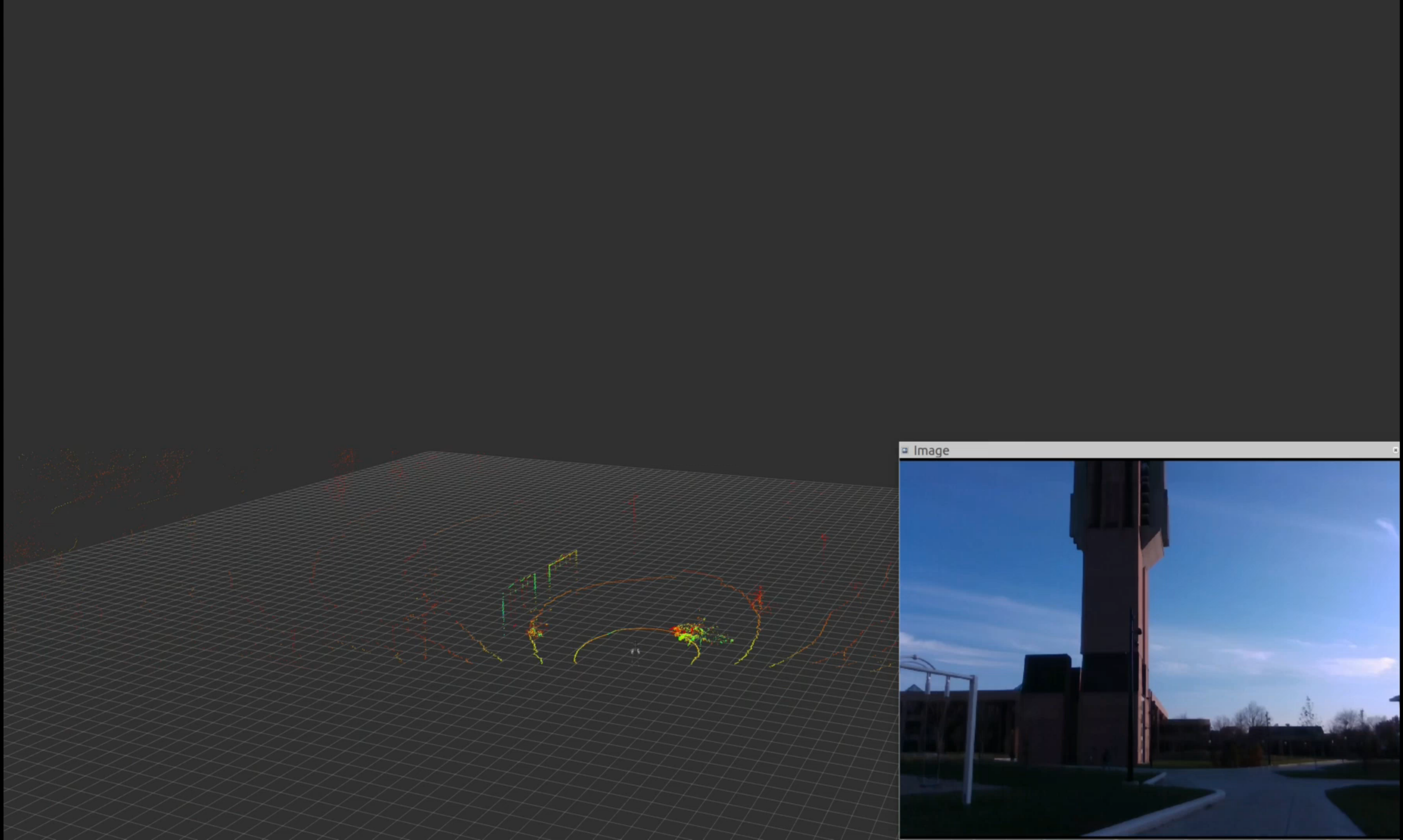
We use the high-frequency odometry from the InEKF to correct for Cassie's movement within single LiDAR scans.



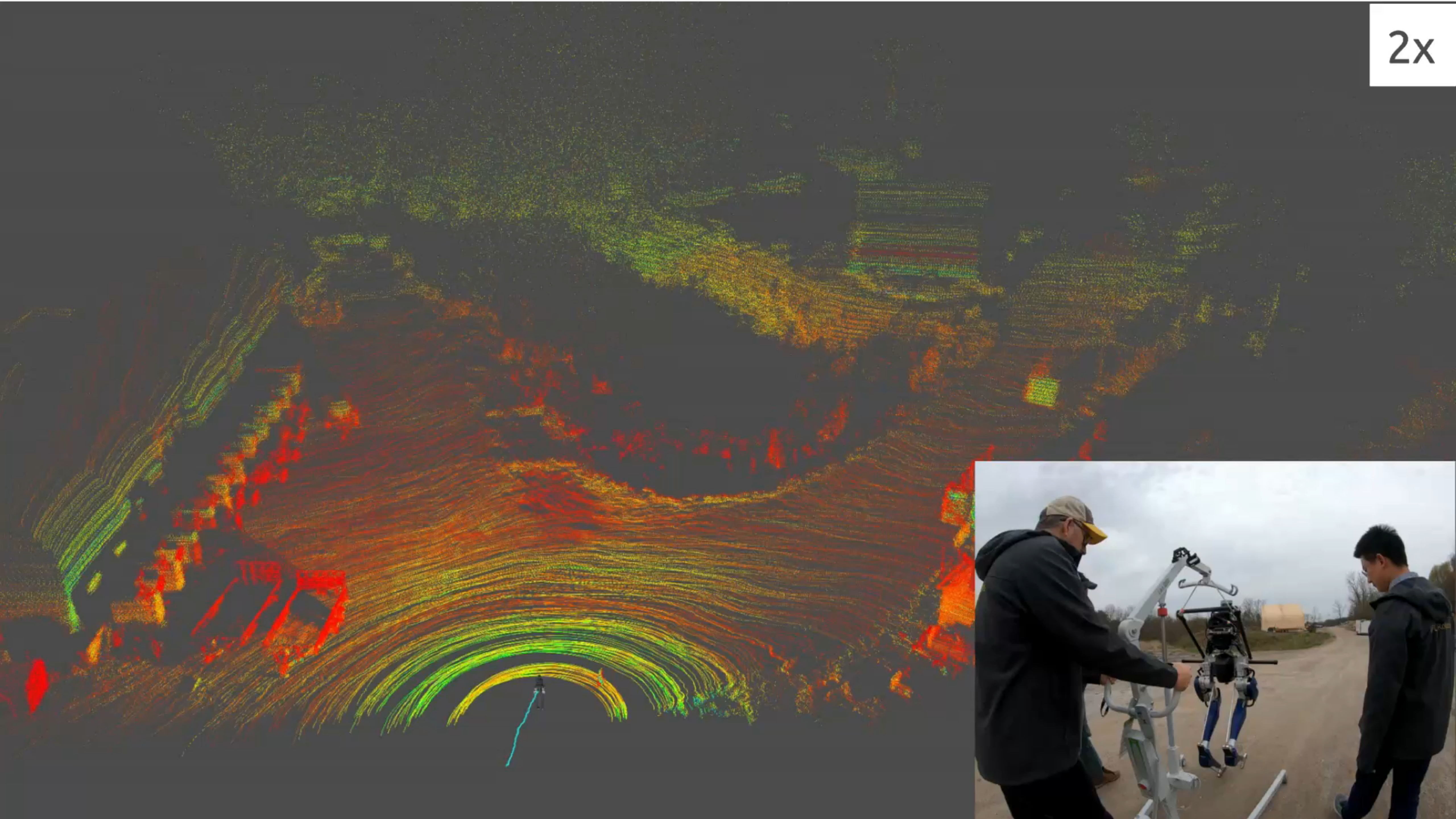
without motion compensation



with motion compensation



Image


2x

InEKF SLAM with Landmarks

$$
\mathbf{X}_t \triangleq
\begin{array}{c}
\begin{array}{ccc}
\overbrace{\hphantom{\mathbf{R}_{\mathrm{WB}}(t) \quad {}_{\mathrm{W}}\mathbf{v}_{\mathrm{WB}}(t) \quad {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WB}}(t)}}^{\text{Orientation, velocity and position of IMU in world frame}}
&
\overbrace{\hphantom{{}_{\mathrm{W}}\mathbf{p}_{\mathrm{WC}_1}(t) \quad \cdots \quad {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WC}_N}(t)}}^{\text{Position of contact points in world frame}}
&
\overbrace{\hphantom{{}_{\mathrm{W}}\mathbf{p}_{\mathrm{WL}_1}(t) \quad \cdots \quad {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WL}_N}(t)}}^{\text{Position of landmarks in world frame}}
\end{array}
\\
\begin{bmatrix}
\mathbf{R}_{\mathrm{WB}}(t) & {}_{\mathrm{W}}\mathbf{v}_{\mathrm{WB}}(t) & {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WB}}(t) & {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WC}_1}(t) & \cdots & {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WC}_N}(t) & {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WL}_1}(t) & \cdots & {}_{\mathrm{W}}\mathbf{p}_{\mathrm{WL}_N}(t) \\
\mathbf{0}_{1,3} & 1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\mathbf{0}_{1,3} & 0 & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\mathbf{0}_{1,3} & 0 & 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\
\mathbf{0}_{1,3} & 0 & 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \\
\mathbf{0}_{1,3} & 0 & 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{0}_{1,3} & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 1
\end{bmatrix}
\end{array}
$$

Additional Types of Invariant Measurements

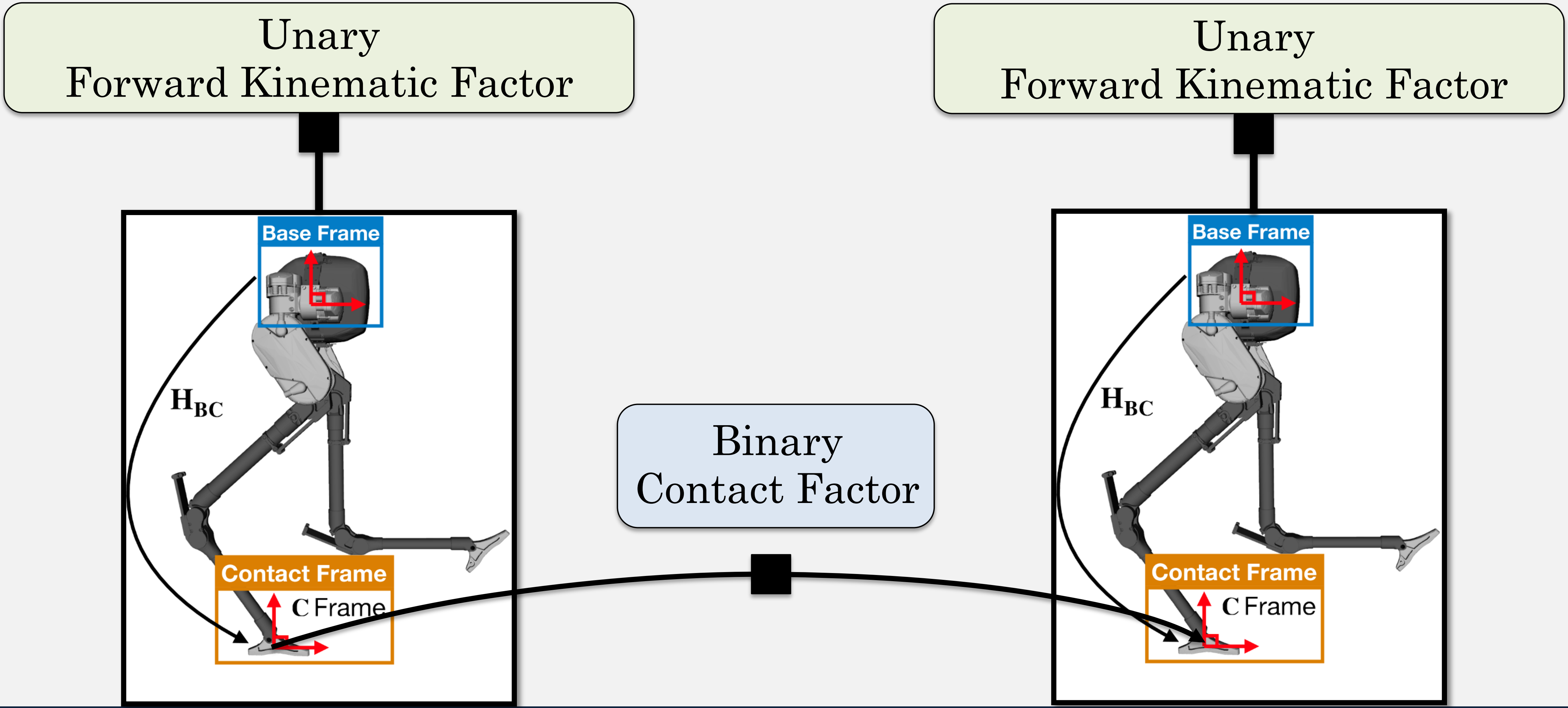
- Landmark Measurement (right invariant) [Zhang 2017]
- GPS Measurement (left invariant) [Barczyk 2011] [Barrau 2015]
- Magnetometer Measurement (right invariant) [Barczyk 2011] [Barrau 2015]
- Position, Velocity, or Pose Measurement (right or left invariant)

Open source C++ library
(https://github.com/RossHartley/invariant-ekf)

Extendable to many aided-inertial navigation systems (wheeled or flying robots!)

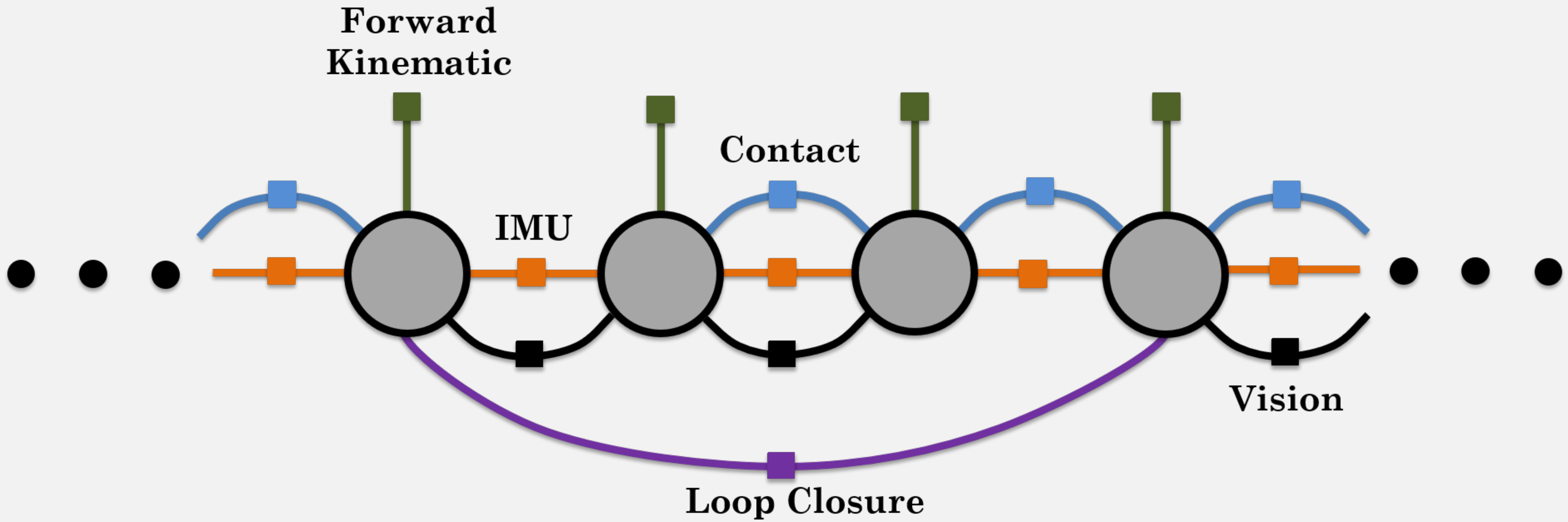
The pose of the robot
is estimated from
Invariant EKF odometry
in the IMU frame

Extends to Factors Graphs



[Hartley et al. IROS 2018]

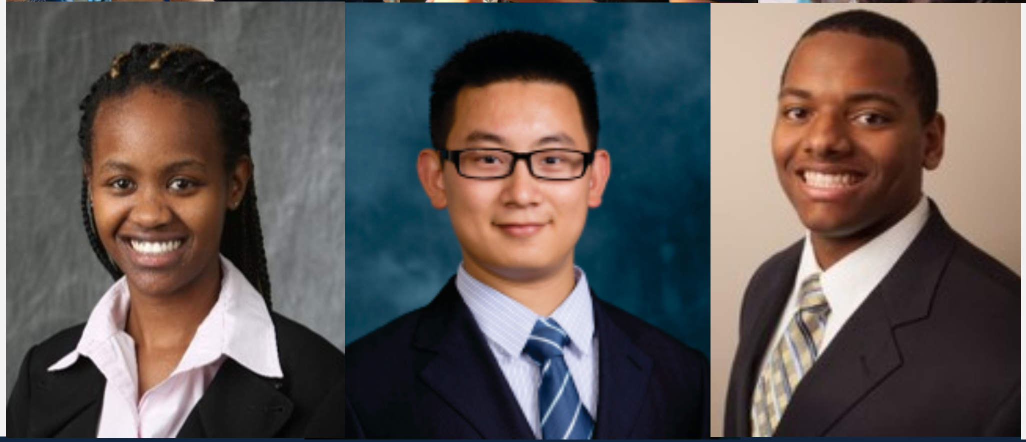
Visual-Inertial-Contact Factor Graph



$$\underset{\mathcal{X}_k}{\text{minimize}} \ \|\mathbf{r}_0\|^2_{\boldsymbol{\Sigma}_0} + \sum_{i,j \in \mathcal{K}_k} \|\mathbf{r}_{\mathcal{L}_{ij}}\|^2_{\boldsymbol{\Sigma}_{\mathcal{L}_{ij}}} + \sum_{i,j \in \mathcal{K}_k} \|\mathbf{r}_{\mathcal{I}_{ij}}\|^2_{\boldsymbol{\Sigma}_{\mathcal{I}_{ij}}} + \sum_{i,j \in \mathcal{K}_k} \|\mathbf{r}_{\mathcal{V}_{ij}}\|^2_{\boldsymbol{\Sigma}_{\mathcal{V}_{ij}}} + \boxed{\sum_{i \in \mathcal{K}_k} \|\mathbf{r}_{\mathcal{F}_i}\|^2_{\boldsymbol{\Sigma}_{\mathcal{F}_i}}} + \boxed{\sum_{i,j \in \mathcal{K}_k} \|\mathbf{r}_{\mathcal{C}_{ij}}\|^2_{\boldsymbol{\Sigma}_{\mathcal{C}_{ij}}}}$$

Prior — **Loop closure** — **IMU** — **Visual** — **Forward Kinematic** — **Hybrid Contact**

[Hartley et al. IROS 2018]

Thank you!

Questions?