Problem set 2

This problem set is due on October 6 (Friday) at 6pm. Drop your assignment into the drop-box located in front of my office door (Purnell 413). No late homework will be accepted.

I. Search and Unemployment

A. Two-sided model (DMP model)

1) (5pts) Given a share of worker’s bargaining power \(a\), a matched firm and a worker determine how to split the total revenue, which is \(z - b\). The worker’s portion of TS is set to be worker’s surplus: \(a(z - b) = w - b\), through which the wage \(w\) is set to \(az + (1 - a)b\).

2) (5pts) This is the case of a decrease in \(k\) (cost of recruitment): \(j \uparrow, Q \uparrow, u \downarrow, v \uparrow, Y \uparrow, w\) no change, TS no change. See the figures below.

3) (5pts) Two cases are acceptable for answers: ① a decrease in \(k\), ② an increase in the firm’s profit (and/or total surplus). If you choose ①, the results are the same as in (2). If you choose ②, the results are as follows: \(j \uparrow, Q \uparrow, u \downarrow, v \uparrow, Y \uparrow, w \uparrow, TS \uparrow\).

4) (5pts) There are two possible equilibria in the Keynesian DMP model. Here, I use the case where \(w_1 > w^*\) such that \(w^*\) is the socially optimal wages and \(w_1\) is the equilibrium wage in the model. The underlying assumption of this case is that \(Q_1 < Q^*\). The model predicts that ① \(j_1\) is lower than \(j^*\), ② \(u_1\) is higher than \(u^*\), ③ \(v_1\) is lower than \(v^*\), ④ \(Y_1\) is lower than \(Y^*\). ② and ③ indicate the Beveridge curve relationship!!

5) (5pts) For example, an increase in the labor market uncertainty, a bigger labor market, etc..

6) (10pts) Suppose that \(w\) is “too high” in eq’l’m, which implies that \(Q\) and \(j\) are too low relative to what is socially efficient. If the gov’t were to subsidize successful matches by
pumping ‘s’ to a firm when a match occurs, the firm’s cond’n becomes \( em(\frac{1}{j}, 1) = \frac{k}{z-w+s} \), that is, the firm’s profit increases. If the gov’t pays subsidies that offset the difference between \( w^H \) (eq'l m wage in the Keynesian DMP) and \( w^* \) (efficient wage), \( j^H \) becomes to closer to \( j^* \). So does \( Q^H \). Eventually, it corrects the problem. Note that you can think of subsidizing \( s \) as a decrease in the cost of posting a vacancy. In that case, \( em(\frac{1}{j}, 1) = \frac{k-s}{z-w} \). But the results should be the same as the previous case where subsidizing \( s \) is considered as an increase in the firm’s profit.

B. Efficiency wage model

(5pts) The worker’s effort function shifts down causing an increase in the efficiency wage from \( w_1 \) to \( w_2 \).

II. Growth Model

1) (5pts) Initial eq'l m=A, transitional change=A_t, new eq'l m=A_N.

In transition, \( c \) falls and \( \frac{N'}{N} \) starts to fall. Due to a decrease in the pop’n, \( c \) starts to increase up to the original eq'l m \( c \), and the pop’n grows up to 1. Thus, in eq'l m, \( c \) remains the same as before while \( l \) increases. Note that the pop’n size increases while the growth rate of pop’n is 1 in eq'l m. (refer to the figures below.)
2) (5pts) Initial eq'l'm=\(A\), new eq'l'm=\(A_N\).

A decrease in the death rate shifts up the function \(g(c)\) which leads to an increase in the size of pop'n. Eventually, \(c\) and \(l\) both decrease.

3) a) (5pts) Start from the LMK:

\[ K' = (1 - \delta)K + sY \]
\[ \Leftrightarrow \frac{K' N'}{N'} = (1 - \delta) \frac{K}{N} + s \frac{Y}{N} \]
\[ \Leftrightarrow k' = \frac{1}{1 + n} \left[ (1 - \delta)k + szk^\alpha \right]. \]

b) (5pts)

\[ \% \Delta k = \frac{k' - k}{k} = \frac{1}{1 + n} \left[ (1 - \delta) + szk^{\alpha - 1} \right] - 1. \]
c) (2.5pts) Using the results in (b), \( \frac{\Delta GR_k}{\Delta z} > 0 \).

d) (2.5pts) Using the results in (b), \( \frac{\Delta GR_k}{\Delta n} > 0 \).

e) (5pts) Using the eqm condition, \( k^* = (\frac{sz}{n+\delta})^{1-\alpha} \). Thus, \( \frac{\Delta k^*}{\Delta z} > 0 \).

f) (2.5pts) \( \frac{\Delta k^*}{\Delta n} > 0 \).

g) (2.5pts) \%\Delta Y^* = n, and \%\Delta y^* = 0

h) (2.5pts) \( y^* = z(k^*)^{\alpha}, \ c^* = (1-s)z(k^*)^{\alpha}, \ i^* = sz(k^*)^{\alpha}, \ r^* = \alpha z(k^*)^{\alpha-1}, \ w^* = (1-\alpha)z(k^*)^{\alpha} \).

4) a) (10pts) As always, start from the LMK,

\[
K' = (1-\delta)K + szF(K,bN) \\
\Leftrightarrow \frac{K'}{bN'} \frac{bN}{bN} = (1-\delta) \frac{K}{bN} + szF(\frac{K}{bN}, 1) \\
\Leftrightarrow \tilde{k}' = \frac{1}{(1+n)(1+f)}[(1-\delta)\tilde{k} + szf(\tilde{k})]
\]

where \( \tilde{k} \) is per efficiency unit of capital \( (\frac{k}{bN}) \), \( f(\tilde{k}) = F(\tilde{k}, 1) \). In SS, \( \tilde{k} = \tilde{k}' = k^{**} \), and \( k^{**} \) is constant!!

\[ \%\Delta K^* = n + f, \text{ and } \%\Delta k^* = f \]

b) (2.5pts) \( \frac{\Delta GBk^*}{\Delta y} > 0 \).

5) a) (2.5pts) \( y_p = z_p u H_p \) (per capita income in the poor country), \( y_r = z_r u H_r \) (per capita income in the rich country).

b) (2.5pts) \( \%\Delta y_p = b(1-u), \%\Delta y_r = b(1-u) \).

c) (5pts) The gap in \( y \) remains constant in the SS (no convergence) in the endogenous growth model, while the gap between the two countries with different initial \( y \) does converge.