1 Regular languages

A language is regular iff there is a rewrite grammar which generates it whose production rules all have one of the following forms

\[ A \rightarrow \lambda \]
\[ A \rightarrow aB \]

where \( A \) and \( B \) are non-terminal symbols and \( a \) is any terminal symbol.

**Example 1** The language \( L = \{cv, cvcv, cvcvcv, \ldots\} \) is regular.

**Exercise 1** Can you write a rewrite grammar which accepts language

\[ L = \{cv, cvcv, cvcvcv, \ldots\} \]

whose production rules have the required form above?

Another way of representing these grammars is with a diagram. The nonterminal symbols represent the “states” of the diagram, indicated by circles. Production rules of the form \( A \rightarrow aB \) are indicated by transitions (arrows) from state \( A \) to state \( B \) with the label \( a \). Production rules of the form \( A \rightarrow \lambda \) are indicated by marking a state with double peripheries. These states are called “final” states because that is where the generation process ends. The starting category is indicated with an incoming arrow beginning from nowhere.

**Example 2** The finite-state diagram for \( L = \{cv, cvcv, cvcvcv, \ldots\} \) is shown here.

These diagrams are also called finite-state diagrams. Remember, there are only finitely-many terminal symbols and so under the translation we described there are only finitely many states. The diagrams are also called finite-state machines, and the particular machine above is called a finite-state acceptor because it recognizes/generates/accepts a language (as opposed to doing something else).

The way we can see what the finite-state acceptor does is to

**Exercise 2** Draw a finite-state diagram that describes the SP language where the subsequences [fs, sf] are forbidden.

**Exercise 3** Draw a finite-state diagram that describes a language where there must be an even number of [f] sounds.

---

1Next week we will examine finite-state transducers and weighted finite-state machines.
Exercise 4 Is the language where words an even number of $[f]$ sounds Strictly $k$-local for any $k$? Strictly $k$-Piecewise for any $k$?

Exercise 5 Try to draw a finite-state diagram for the language $a^n b^n$. What problems do you encounter?