Solution to Exercise C-3.14, Goodrich & Tamassia, Page 214

Oct 11, 2001

**R-3.14:** Let $T$ and $U$ be 2-3-4 trees storing $n$ and $m$ items, respectively, such that all the items in $T$ have keys less than the keys of all the items in $U$. Describe a method requiring time dominated by $\log n + \log m$ for joining $T$ and $U$ (destroying the old versions of $T$ and $U$).

**Answer:** We outline a method for accomplishing this task.

1. Find the height, $h_T$, of tree $T$ by going down the right-most side of $T$, splitting any 4-nodes found on this path. Splitting 4-nodes may make the original height of the tree increase. If this happens, set $h_T$ to the new height.
   
   This step takes time codominant with $\log n$.

2. Find the height (after any splitting), $h_U$, of $U$ by going down the left-most side of $U$, splitting any 4-nodes found on this path.
   
   This takes time codominate with $\log m$.

3. Suppose $h_T < h_U$ (there is an analogous case if $h_T > h_U$).
   
   Delete the right-most item, $r$, of $T$, that is, the item with the largest key. This may cause the height of $T$ to decease by 1. If so, update $h_T$.
   
   This requires time codominant with $\log n$.

4. Make the left-most node of $U$ (which will be a 2-node or 3-node because of the splitting of 4-nodes done in step (2)) that appears at one level above the root of $T$ (when the external nodes of both trees are placed at the same level) into a a 3- or 4-node by inserting item $r$ into this node and letting key $k_1$ of this node be the key of item $r$ and the left subtree of the node be $T$. This is now the required 2-3-4 tree.
   
   The time for this step is dominated by $\log m$.

5. If $h_T = h_U$, we delete the right-most item, $r$, of $T$. If the height of $T$ decreases with this deletion, then go to step (4). Otherwise, join the trees by making a new 2-node containing $r$ as the root with $T$ and $U$ being the left- and right-subtrees, respectively. This is now the required 2-3-4 tree.
   
   This can be done in time codominant with $\log n$. 

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This procedure will be illustrated with an example in lecture.

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