Solution to Exercise R-1.9, Page 48

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**R-1.9** Let $T(n)$ be defined by the recurrence equation

$$
T(n) = \begin{cases} 
4 & \text{if } n = 1 \\
T(n-1) + 4 & \text{for } n > 1.
\end{cases}
$$

Show, by induction, that $T(n) = 4n$.

**Answer** Base case $n = 1$: $T(1) = 4$ by definition and $4n = 4$ when $n = 1$. Thus $T(n) = 4n$ when $n = 1$.

**Induction step** Induction hypothesis: Assume $T(k) = 4k$ for some arbitrary integer $k \geq 1$. Then we will show that it must hold for the next integer, $k + 1$, as well.

By definition $T(k + 1) = T(k) + 4$. By the induction hypothesis, $T(k) = 4k$, thus $T(k + 1) = 4k + 4 = 4(k + 1)$. Hence, the result holds for $n = k + 1$ and by induction must therefore hold for all $n \geq 1$. 

BFC