**Theorem 5.6 [The Master Theorem]:** Let

\[
T(n) = \begin{cases} 
  c & \text{if } n < d, \\
  aT(n/b) + f(n) & \text{if } n \geq d,
\end{cases}
\]

where \(a \geq 1, b > 1,\) and \(d\) are integers and \(c\) is a positive constant.

1. If there is a small constant \(\epsilon > 0,\) such that \(f(n) \leq n^{\log_b a - \epsilon},\) then \(T(n) \sim n^{\log_b a}.
2. If there is a constant \(k \geq 0,\) such that \(f(n) \sim n^{\log_b a \log k n},\) then \(T(n) \sim n^{\log_b a \log^{k+1} n}.
3. If there are small constants \(\epsilon > 0\) and \(\delta < 1,\) such that \(f(n) \geq n^{\log_b a + \epsilon}\) and \(af(n/b) \leq \delta f(n),\) for \(n \geq d,\) then \(T(n) \sim f(n).

Note the key roles that are played by the constants \(a\) and \(b\) along with the function \(f(n)\) in the above theorem. The constants \(c\) and \(d\) are lesser importance in that they do not enter the conclusions except in case (3) where \(d\) enters. Case (2) is the one that we will use the most.

**Example.** Let

\[
T(n) = \begin{cases} 
  c & \text{if } n = 1, \\
  2T(n/2) + dn & \text{for some constant } d > 0.
\end{cases}
\]

Then \(T(n) \sim n \log n\) by part (2) of the master theorem.

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1 From page 268 of Goodrich and Tamassia.