LEARNING TOOLS

The manner in which students learn mathematics influences how well they understand its concepts, principles, and practices. Many researchers have argued that to promote learning with understanding, mathematics educators must consider the tasks, problem-solving situations, and tools used to represent mathematical ideas. Mathematical tools foster learning at many levels—namely, the learning of facts, procedures, and concepts. Tools can also provide concrete models of abstract ideas, or, when dealing with complex problems, they can enable students to manipulate and think about ideas, thereby making mathematics accessible and more deeply understood.

Mathematical learning tools can be traditional, technological, or social. The most frequently employed tools are traditional, which include physical objects or manipulatives (e.g., cubes), visualization tools (e.g., function diagrams), and paper-and-pencil tasks (e.g., producing a table of values). Technological tools, such as calculators (i.e., algebraic and graphic) and computers (e.g., computation and multiple-representation software), have gained attention because they can extend learning in different ways. Social tools, such as small-group discussions where students interact with one another to share and challenge ideas, can be considered a third type of learning tool. These three tools can be used independently or conjointly, depending on the type of learning that is intended.

Learning Tools in Mathematics

A learning tool can be as simple as an image or as complex as a computer-based environment designed to improve mathematical understanding. The key characteristic of a learning tool is that it supports learners in some manner. For example, a tool can aid memory, help students to review their problem-solving processes, or allow students to compare their performance with that of others, thereby supporting self-assessment. Learning tools can represent mathematical ideas in multiple ways, providing flexible alternatives for individuals who differ in terms of learner characteristics. For example, learners who have difficulty understanding the statistical ideas of arithmetic mean (center) and variance (spread) may be assisted through interactive displays that change as data points are manipulated by the learner. A mathematical learning tool can scaffold the learner by performing computations, providing more time for students to test mathematical hypotheses that require reasoning. In the statistics example, learners can focus on why changes to certain parameters affect data—and in what ways, rather than spending all their time calculating measures.

Traditional Tools. Traditional tools are best suited for facilitating students’ learning of basic knowledge and skills. Objects that can be manipulated, such as cubes, reduce the abstract nature of concepts, such as numbers, thereby making them real and tangible, particularly for younger children. Such tools support the development of children’s understanding of arithmetic by serving as a foundation for learning more complex concepts. Visualization tools, such as graphs, can support data interpretation, while paper-and-pencil tools that provide practice of computational skills can support memory for procedures and an ability to manipulate symbols. Combining physical tools with visualization tools can substantially increase students’ conceptual knowledge. Dice and spinners, for example, can be used to support elementary school students in creating graphs of probability distributions, helping them develop an understanding of central tendency.

Technological Tools. Technological tools are most effective in facilitating students’ understanding of complex concepts and principles. Computations and graphs can be produced quickly, giving students more time to consider why a particular result was obtained. This support allows students to think more deeply about the mathematics they are learning. Electronic tools are necessary in mathematics because they support the following processes: (a) conjectures—which provide access to more examples and representational formats than is possible by hand; (b) visual reasoning—which provides access to powerful visual models that students often do not create for themselves; (c) conceptualization and
modeling—which provide quick and efficient execution of procedures; and (d) flexible thinking—which support the presentation of multiple perspectives.

Spreadsheets, calculators, and dynamic environments are sophisticated learning tools. These tools support interpretation and the rapid testing of conjectures. Technology enables students to focus on the structure of the data and to think about what the data mean, thereby facilitating an overall understanding of a concept (e.g., function). The graphics calculator supports procedures involving functions and students' ability to translate and understand the relationship between numeric, algebraic, and graphical representations. Transforming graphical information in different ways focuses attention on scale changes and can help students see relationships if the appropriate viewing dimensions are used. Computers may remove the need for overlearning routine procedures since they can perform the task of computing the procedures. It is still debatable whether overlearning of facts helps or hinders deeper understanding and use of mathematics. Technology tools can also be designed to help students link critical steps in procedures with abstract symbols to representations that give them meaning.

Video is a dynamic and interactive learning tool. One advantage of video is that complex problems can be presented to students in a richer and more realistic way, compared to standard word problems. An example is The Adventures of Jasper Woodbury, developed by the Learning Technology Center at Vanderbilt University. Students are required to solve problems encountered by characters in the Woodbury video by taking many steps to find a solution. This tool supports students' ability to solve problems, specifically their ability to identify and formulate a problem, to generate subgoals that lead to the solution, and to find the solution. However, the information presented in a video cannot be directly manipulated in the same way that data can be changed in spreadsheets and calculators.

Learning tools that present the same information in several ways (e.g., verbal equation, tabular, graphic) are referred to as multiple-representation tools. The ability to interpret multiple representations is critical to mathematical learning. There is evidence to suggest that multiple representations can facilitate students' ability to understand and solve word problems in functions, and to translate words into tables and graphs. However, interpretation is not easy without some kind of support. One type of support involves highlighting common elements between the different representations to make the relationship between each explicit, thereby facilitating interpretation in both contexts. In some cases, this type of support is insufficient and students need to be explicitly taught to make the connections. Multiple representations can be a powerful learning tool for difficult problems—when students have acquired a strong knowledge base.

Additional research is needed to determine the exact benefits of multiple representational tools. It is important to emphasize that, as with any educational innovation, mathematical learning tools must be designed with a consideration of the teacher, curriculum, and student in mind. For example, with the help of curricular teams and teachers, complex computer environments that present students with multiple representation tools for learning algebra and geometry were successfully adopted in several school systems in the United States.

Social tools. Social tools are a fairly recent consideration. In the 1990s, small-group work where students share strategies for solving problems began to be used as a powerful learning tool. This tool facilitates students' ability to solve word problems and to understand arithmetic. Group collaboration while learning with technology can help students develop the perspectives and practices of mathematics, such as what constitutes acceptable mathematical evidence. Peers and computers can provide feedback that makes students aware of contradictions in their thinking. In this way, social tools can assist learning and transform understanding.

Issues for Further Consideration

Mathematical learning tools should be an important part of students' educational experience. However, a few issues must be addressed before their potential is fully realized. First, use of technological tools is fairly limited in classrooms, despite their potential in changing the nature of mathematical learning. Moreover, software used in schools is often geared towards the practice of computational skills. For example, there may be a potential misuse of the graphing calculator if it is not utilized in the context of sense-making activities. There is a fine line between using a tool for understanding and using it because problems cannot be solved without its use.

Second, learning tools should be an integral part of instructional activities and assessment tasks.
Learning tools should be a regular part of the mathematics experience at every educational level, and different tools should be used for various purposes. The question of ethics and equity is raised when technological tools that are used in instruction are not accessible in assessment situations.

Third, learning tools will only meet their promise through professional development. Teachers who understand the strengths and weaknesses of tools can have a strong impact on how they are used. Support is needed at all levels of education to ensure that sophisticated learning tools are available for use in every mathematics classroom. Learning tools are only as good as the activities that provide the mathematical experiences. The effectiveness of such tools is thus highly dependent on the purpose of the activity and the learning that is intended.

See also: Mathematics Learning, subentry on Complex Problem Solving; Science Learning, subentry on Tools; Technology in Education, subentry on Current Trends.

BIBLIOGRAPHY


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MYTHS, MYSTERIES, AND REALITIES

According to the National Research Council, "Much of the failure in school mathematics is due to a tradition of teaching that is inappropriate to the way most students learn" (p. 6). Yet, despite the fact that numerous scientific studies have shown that traditional methods of teaching mathematics are ineffective, and despite professional recommendations for fundamental changes in mathematics curricula and teaching, traditional methods of teaching continue. Indeed, mathematics teaching in the United States has changed little since the mid-twentieth century—essentially, teachers demonstrate, while students memorize and imitate.

Realities

Although research indicates that learning that emphasizes sense-making and understanding produces a better transfer of learning to new situations, traditional classroom instruction emphasizes imitation and memorization. Even when traditional instruction attempts to promote understanding, most students fail to make sense of the ideas because classroom derivations and justifications are too formal and abstract. Though research indicates that mathematical knowledge is truly understood and us-