A note on welfare measures

Jorge Soares*
George Washington University
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Abstract

I show that the “consumption variation” approach, commonly used in infinitely lived agent models to measure the welfare impact of a policy, is not accurate in overlapping generations environments. While in an infinitely lived agent environment the consumption variation can be shown to be equivalent to a measure based on the Hicks compensation principle, this is not true in overlapping generations economies.

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*Department of Economics, 2201 G St. NW, George Washington University, Washington, DC 20052. Tel. (202) 994-0192, E-mail jsoares@gwu.edu.
1. Introduction

A standard approach to evaluate how individuals are affected by a change to the economic environment is to compute the percent change in an agent’s consumption needed for him to attain the new utility level in the initial environment, this is generally called “consumption equivalence variation”.

In macroeconomics, this approach has been mostly applied to evaluate welfare gains or losses generated by policy changes or other exogenous occurrences in applied dynamic general equilibrium models with infinitely lived agent. With the recent resurgence of computational overlapping generations models, many authors have applied the procedure in this framework as its computation is simple and straightforward (see Campbell, Cocco, Gomes and Maenhout (2000), Conesa and Krueger (1999), Krueger and Kubler (2002), and Storesletten, Telmer and Yaron (2000 and 2001), for instance).

Even though, under some general conditions, this measure is an accurate way of assessing the impact of a policy in an economy with infinitely-lived agents, I show in this paper that it might be misleading when applied to an overlapping generations model.

A proportional change in an agent’s optimal consumption levels can place him in an optimal path, the one he would be on if he had more resources and would re-optimize his decisions. In fact, the proportional change in consumption can easily be converted into a welfare measure based on the Hicks compensation principle. However, in a standard overlapping generations model this is not the case. Because of discontinuities in an individual’s labor supply, if an agent is given more resources he does not re-allocate them across time in such a way that his consumption changes proportionally in every period of his life. Hence, measures that are based on this approach can overestimate the welfare gains or losses incurred by individuals upon a change in the economic environment.

2. The Economic Environment

In this paper, I look at two model economies. The first one is a standard infinitely lived agents economy where agents maximize their lifetime discounted utilities subject to the usual budget constraints. In the second economy, an overlapping generations one, agents live only for $T$ periods and in each period, a large number of agents are born.

The “momentary” utility function is a constant relative risk aversion form of a Cobb-Douglas consumption-leisure index, $U(c,l) = \frac{(c^{\sigma}l^{1-\sigma})^{1-\rho}}{\rho}$, where $\rho$ is the inverse of the intertemporal elasticity of substitution, and $\sigma$ is the coefficient of consumption on the Cobb-Douglas index.

The budget constraint facing an individual of age $i$ (age is irrelevant for infinitely lived agents) is described by $a_{i+1,t+1} = (1 + r_{t})a_{i,t} + w_{t}h_{i,t} - c_{i,t}$, where $h_{i,t}$ is the labor supplied by an age $i$ individual before retirement (which never occurs in the standard infinitely-lived agent model), $a_{i,t}$ denotes the asset holdings of an age $i$ individual at the beginning of the period $t$. $w_{t}$ is the real wage rate and $r_{t}$ denotes the rate of return on assets. I assume, an agent will not accumulate any assets in the last period of his life so that for finitely lived agents $a_{T+1,t+1} = 0$, $\forall t$. 
3. A measure of welfare based on preferences

To evaluate the impact of a change in the economic environment on the well-being of the individuals, we need to compare the utility of the individuals in the initial economy to their utility in the economy after the change.

I assume that the economy is in a steady-state with an economic environment described by $S^0$ and I compute the equilibrium for the economy when a change occurs. I then compare the utility of agents in the initial economy with the utility they attain under the new economic environment, $S^1$.

To assess how individuals fare under the new economic environment, I construct a measure of the welfare gains from the change based on the Hicks compensation principle. I compute the level of transfers required to make each individual able to attain the same level of lifetime utility in the welfare gains from the change based on the Hicks compensation principle. I compute the level of individual to consumption and savings.

Both measures are equivalent as we can compute the economy’s steady-state as in the new economy, allowing the transfer to be optimally allocated by the individual to consumption and savings.

The compensation to be given to an age-i agent endowed with a level of assets $a_i$ in an economic environment described by $S^0$ is $x_i$ such that $V_i(a_i + x_i, S^0) = V_i(a_i, S^1)$, where the right-hand side is the lifetime utility level of that agent in the new economy.

The measure of welfare costs is then $z_i = (1 + r) * x_i$ where $r$ is the real interest rate in the reference steady-state.

4. Commonly used measures of welfare

In infinitely lived agent’s models it is usual to compute as a measure of welfare the percent change in consumption necessary to make each agent as well off in the initial economy as in the new economy. That is, the welfare measure is $\epsilon$ such that:

$$
\sum_{t=0}^{\infty} \beta^t U((1 + \epsilon) c_t(a_0, S^0), l_t(a_0, S^0)) = \sum_{t=0}^{\infty} \beta^t U(c_t(S^1), l_t(S^1)).
$$

The optimality conditions for the agent’s problem are: $U_c(c, l_t) = \beta(1 + r_{t+1}) U_c(c_{t+1}, l_{t+1})$ and $U_l(c, l_t) = w_t U_l(c, l_t)$.

For our utility specification $U_c(c, l_t) = \sigma c_t^{(1-\rho)-1} l_t^{(1-\sigma)(1-\rho)}$ and $U_l(c, l_t) = (1 - \sigma) c_t^{(1-\rho)} l_t^{(1-\sigma)(1-\rho)-1}$, and a proportional increase in consumption only would move the agent away from the optimal path. Labor supply is such that: $(1 - \sigma) c_t = \sigma w_t l_t$ so in order to meet the optimality conditions we need to increase consumption and leisure by the same proportion. We can use as welfare measure the percent change in consumption and leisure necessary to make each agent as well off in the initial economy’s steady-state as in the new economy, $\tilde{\epsilon}$. For our utility specification, assuming interior solutions, $\tilde{\epsilon}$ is such that $1 + \tilde{\epsilon} = [V(a_0, S^1)/V(a_0, S^0)]^{1/(1-\rho)}$. In an infinitely lived agent model both measures are equivalent as we can compute $\tilde{\epsilon}$ from $\epsilon$: $1 + \tilde{\epsilon} = (1 + \epsilon)^{\sigma}$.

Once we have $\tilde{\epsilon}$ we can compute the present value of this increase and obtain the Hicksian compensation measure constructed above, $z = \sum_{t=0}^{\infty} \frac{\tilde{\epsilon}^{t+1}}{\prod_{i=0}^{t+1} (1 + r_i)}$, since when we increase consumption and leisure by $\tilde{\epsilon}$ we guarantee that the agent is in an optimal path.

Because of the simplicity in computing this measure, it has been applied in many other frameworks besides the standard infinitely lived agent model. Recently this approach has been applied
to overlapping generations (OG) models. But, these measure, ε or \( \bar{\varepsilon} \), should not be used lightly in overlapping generations models and in frameworks that do not share some crucial characteristics with the standard infinitely lived agents model described previously.

The optimality conditions for an agent in a two-period OG model are:

\[
\begin{align*}
    c_{1,t}^{\sigma(1-\rho)-1}l_{1,t}^{(1-\sigma)(1-\rho)} &= \beta(1 + r_{t+1})c_{2,t+1}^{\sigma(1-\rho)-1}, \\
    (1-\sigma)c_{1,t} &= \sigma w_t l_{1,t} \text{ and } l_{2,t+1} = 1.
\end{align*}
\]

Hence a proportional increase in consumption would meet the Euler equation condition but not the optimality condition for the labor supply decision. Moreover, a proportional increase in leisure before retirement would deliver the labor supply decision, but not the Euler equation. Therefore, it is not possible to compute a measure based on a proportional increase in consumption that meets all the optimality conditions. The present value of the implied transfers would give us a measure that overestimates the welfare gains or losses of the change to the economic environment. That is, if we were to allow for re-optimization, we would need to transfer fewer resources to the agent for him to be as well off in the initial economic environment as in the new one.

5. A Numerical example

To illustrate the potential amplitude of this error, I compute the Hicksian compensation measure and the present value of the transfers associated with the proportional increase in consumption needed to equate the lifetime utilities across environments for a 60 period OG model. I assume that agents work for the first 45 years of their lives and retire thereafter. I assume that factor prices remain fixed and I evaluate how agents fair upon the introduction of a social security system. The model is calibrated as in Soares (2003). Figure 1 shows the bias in terms of initial output of measures based on \( \bar{\varepsilon} \) (‘value function’) and \( \varepsilon \) (‘consumption’) relatively to the Hicksian measure. It also show the bias of a commonly used actuarial fair measure, the present value of net benefits, as a reference. It is clear that the three alternative measures are not very accurate. In this calibration of the model economy, \( \bar{\varepsilon} \) is a fairly accurate measure to evaluate the impact of social security on younger agents but is far off target for older generations. The reverse happens with \( \varepsilon \). I should point out that using a combination of both measures does not solve the problem as the Euler equation will not hold.

6. Concluding Comments

The analysis in this paper abstracts from many factors that might affect the bias of the “consumption equivalence variation” measure in important ways. There are other frameworks, even infinitely lived agent ones, where the “consumption equivalence variation” approach might not generate an accurate measure of the welfare impact of a change to the economic environment. For instance, economies where individuals are subject to changes or make choices that cause discrete variations in their utility such as the ones affecting their employment, schooling or retirement status.

The inclusion of these factors might reinforce the conclusions of this paper. But, the current results do underline the importance of constructing measures of welfare based explicitly on the specification of individuals’ preferences and taking into account the characteristics of the economic environment under analysis. Ignoring such factors by applying generically a specific measure might lead to erroneous evaluations of how changes in the environment impact on individuals’ well-being.
Figure 1: Bias of alternative welfare measures

References


