Borrowing Constraints, Parental Altruism and Welfare*

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Abstract

This paper investigates the impact of borrowing constraints on welfare in a standard overlapping-generations model where parental altruism results in transfers. I find that the average level of welfare is higher when children cannot borrow against future income. As Bernheim (1989) showed, the Nash-Cournot equilibrium does not maximize the average level of utility of currently living agents; the presence of a borrowing constraint increases children’s savings and parental transfers bringing their levels closer to the optimum, raising children’s welfare as well as average welfare in the short-run and in the long-run.

Additionally, borrowing constraints reduce investment on children’s education, decreasing the aggregate level of human capital, but raises aggregate savings and, hence, physical capital. When prices are flexible, the latter effect dominates and the positive welfare impact of the credit constraint is higher.

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1. Introduction

In this paper, I show that in a standard overlapping generations economy where parents care about the lifetime utility of their children, a borrowing constraint that does not allow children to borrow against future income can be welfare improving both in the long-run and in the short-run.

Credit constraints have been a long time concern of policy makers and economic analysts as they are viewed as a critical obstacle to an efficient allocation of resources. In recent years, the impact of borrowing constraints on human capital accumulation has drawn greater attention in the literature. Typically, children cannot borrow against their future income to finance education, and it is argued that this borrowing constraint prevents children from acquiring optimal levels of consumption and education and makes them worse off. The overall presumption is that the average levels of welfare are lower in the constrained equilibrium than in an unconstrained one.

Additionally, the current debate on the impact of credit constraints has also focused on the relationship between credit constraints and child labor. A growing theoretical literature points to the lack of access to credit as the principal factor behind inefficiently high levels of child labor (e.g., Baland and Robinson, 2000 and Ranjan, 2001), and empirical evidence has found a strong relation between the existence of borrowing constraints and child labor (e.g., Beegle, Dehejia and Gatti, 2003 and Edmonds, 2004). Besides impeding the access of children to resources that they can allocate to education, credit constraints also reduce the time available to education by requiring children to supply inefficiently high levels of labor to finance consumption and the accumulation of human capital.

These alleged harmful impacts of credit constraints have generated support for governments to intervene by developing credit markets or implementing public policies that mitigate the impacts. However, a more straightforward implication of the presumptions on borrowing constraints is that we should set up policies that replicate, in a constrained economy, the allocation of resources that would be obtained with complete markets. For example, Becker and Murphy (1988) argue that welfare policies should replicate social arrangements within the extended family or small communities that in the past implemented the allocation of resources that would be obtained in the presence of complete markets. In the same line, Rangel (2003) and Boldrin and Montes (2005) show that public funding of education and social security policies can be set up to mimic
the unconstrained equilibrium. The support for government policies that help deliver allocations
closer to the unconstrained equilibrium is predicated on the idea that under liquidity constraints
competitive equilibria are inefficient and are “worse” than the unconstrained equilibria.

A clearer understanding of the effect of borrowing constraints is critical to evaluate the impact
that suggested policies might have on the well-being of agents, as the implementation of policies
based on incorrect assumptions can make agents significantly worse off. But, although credit
constraints are central to an individual’s optimal allocation of resources across time, there has not
been a significant amount of formal economic analysis to assess their welfare implications.

I revisit the discussion of the impact of credit constraints on welfare and human capital accu-
mulation in a general equilibrium overlapping generations model with parental altruism. I assume
that children are economic agents with the same preferences as adults, allocating resources to the
acquisition of human capital and enjoying consumption and leisure. Furthermore, I assume that
parents care about their children’s well-being in the sense that children’s lifetime utility enters their
parents’ utility function, and as rational and forward-looking agents, they take into account the
full impact of their decisions on their children’s future income levels.

Surprisingly, both current and future children are made better off by the introduction of a
borrowing constraint in an environment where children borrow against future income to finance
their consumption and education. Furthermore, allowing for the borrowing constraint is welfare
improving in the sense that it increases the average level of welfare in each period henceforth.

The results point in the direction opposite to the common perception in the literature as children
are made better off with the implementation of a borrowing constraint, and the average welfare
levels of agents increases. In an unconstrained economy, if children are Cournot players and have
no strategic power over their parents, the outcome of the game played between them and their
altruistic parents maximizes parents’ utility. However, the resulting Cournot-Nash equilibrium
does not maximize the average level of utility of currently living agents, as was shown in Bernheim
(1989), nor the welfare of children. In fact, if given some strategic power, children would alter
their decisions to take advantage of the positive externality they have on their parents’ utility and
generate a higher level of parental transfers. They would increase savings, reducing consumption,
raising their marginal utility of consumption; parents would respond by increasing transfers to
children. Therefore, the level of parental transfers that maximizes average welfare is also higher
than the level that is optimal for parents to give in the Cournot-Nash equilibrium. By reducing the amount of borrowing by children, a binding credit constraint effectively increases children’s savings and places them at a point in their parents’ reaction function that results in a higher level of parental transfers. By inducing an increase in parental transfers, the borrowing constraint moves the economy closer to the optimum generating aggregate welfare gains. Children are made better off because they receive more transfers and do not have any debt to repay in the future; they move closer to the levels of parental transfers and savings that maximize their welfare. Parents are made worse off because of the decrease in consumption and leisure implied by the increase in parental transfers; however, the increase in their descendant’s life-time utility off-sets this effect, fully or partially.

In addition, the long-run increase in welfare is higher than the one observed in the short-run. Upon the introduction of the borrowing constraint, children’s savings increases while their accumulation of human capital decreases. The first effect dominates, making future parents wealthier. Because wealthier parents are willing to transfer more resources to their children, future children receive more parental transfers and are better off than current ones.

Hence, the widely held presumption that imposing a borrowing constraint leads to lower levels of welfare, at least in the short-run, occurs because some important implications of the borrowing constraint on the outcome of the game played between children and their altruistic parents have been ignored.

This feature has been overlooked in the literature in part because the focus has been on studying the impact of borrowing constraints in infinitely lived agents economies (Aiyagari, 1994) or overlapping generations economies where agents are selfish. Moreover, the emphasis has mostly been on the general equilibrium effects resulting from the impact of borrowing constraints on the accumulation of capital. Jappelli and Pagano (1994) show that by increasing the levels of physical capital borrowing constraints can enhance growth. De Gregorio (1996), Christous (2001), and Buiter and Kletzer (1992) have looked at their impact in overlapping generations models where agents also accumulate human capital. But in these papers agents are selfish which precludes any role for parental transfers. Aiyagari, Greenwood, and Seshadri (2002) study the importance of asset markets on the accumulation of human capital in a world with parental altruism where children
differ by ability. However, they do not account for the impact on children’s welfare.\footnote{Aiyagari, Greenwood, and Seshadri (2002) prove that the introduction of a borrowing constraint decreases the average level of welfare of currently living agents. But in their model children are not active agents which leads the authors to account only for the welfare of adults when they compute their measure of average welfare.} Also in an overlapping generations model with altruism, Altig and Davis (1989) have found that borrowing constraints increase welfare in the long-run. However, they consider only long-run stationary equilibria and attribute the welfare gains to the pecuniary effects of borrowing constraints, namely to the increase in wages generated by the rise in capital resulting from the forced increase in savings. Laitner (1993) looks at the frequency of binding borrowing constraints over the life-cycle in the long-run. He notices that borrowing constraints result in larger parental transfers which mitigate the long-run negative welfare impact of binding borrowing constraints by reducing their frequency. In a different context, Schreft (1992) shows that credit controls can generate a Pareto superior allocation in an environment where individuals make excessive use of credit as a substitute for money in exchange.

In this paper I also account for the impact that borrowing constraints have on welfare through their effect on the aggregate levels of physical and human capital. On the one hand, the inability to borrow against future income hinders children’s investment in education, decreasing the aggregate level of human capital. On the other hand, borrowing constraints can increase aggregate savings and, hence, the aggregate level of physical capital. In the calibrated general equilibrium version of the model, where these changes impact factor prices, the physical capital effect dominates, and amplifies the welfare gains of introducing borrowing constraints.

The paper is organized as follows. In section 2, I build a simple model to analyze the impact of an increase in savings on parental transfers and welfare, and present results that indicate that a borrowing constraint might increase the average level of welfare. In section 3, I present an extended and more realistic economic environment where children borrow against future income to finance consumption and education. Because it is not possible to provide analytical results, I solve the model numerically to assess the impact of a borrowing constraint on welfare. In section 4, the parameters of the economy are calibrated to match long-run features of the US economy. Section 5 presents and analyzes the alternative equilibria. Finally, section 6 concludes and suggests some directions for future research.
2. Some preliminary analytical results

Before analyzing the impact of a borrowing constraint on welfare, it is useful to build a simple model to focus on some features of the overlapping generations environment with parental altruism that are crucial to the role of the borrowing constraint. I start by looking at how the interaction between parents and children determines the levels of parental transfers in the unconstrained economy and its welfare implications. I then establish an important link between children’s savings, parental transfers, and welfare and show that a small constraint on children’s borrowing increases children’s lifetime utility and the average level of welfare.

2.1. A simple model with altruism

To derive analytical results, I study a three period economy where two types of agents live in the two first periods. I assume that an age-1 agent, the child, is born in each of the first two periods. An agent lives for two periods, first as a child then as an adult. As an adult, at age-2, an agent becomes a parent, except in the last period of the economy, and derives utility from her own consumption and also from the child’s lifetime utility.\(^2\)

A child born in period \(t\) \((t = 1, 2)\) maximizes her discounted lifetime utility given by

\[
V_{1,t} = U(c_{1,t}) + \beta U(c_{2,t+1}) + \beta \beta_p V_{1,t+1}
\]  

(2.1)

where \(V_{1,t}\) is the discounted lifetime utility of a child in period \(t\), \(\beta > 0\) is the intertemporal discount factor and \(\beta_p > 0\) is the altruism discount factor, the factor at which the parent discounts the child’s lifetime utility. \(U(\cdot)\) is the utility function which is assumed to be strictly increasing, strictly concave, twice continuously differentiable and to satisfy the Inada conditions. \(c_{i,t}\) is consumption of an \(age - i\) individual in period \(t\). The third period adult has no children, so \(V_{1,3} = 0\).

The first period adult maximizes her discounted lifetime utility given by

\[
V_{2,1} = U(c_{2,1}) + \beta_p V_{1,1}
\]  

(2.2)

\(^2\)I abstract from “two-sided” altruism to simplify the analysis and focus on particular aspects of the interaction between adult parents and their underage children. The largest part of intergenerational transfers, including expenditures on health and education, are from parents to children, moreover, the bulk of the interaction between parents and their offspring occurs when parents are adults and their offspring are children, when children’s altruism might not be relevant. I later show that this assumption is not crucial.
Individuals earn \( y_i \) at age \( i \) as labor income and accumulate assets. The budget constraints facing individuals at time \( t \) can be written as

\[
\begin{align*}
    c_{1,t} &= g_{2,t} + y_1 - a_{2,t+1}, \text{ for } t = 1, 2, \\
    c_{2,t} &= (1 + r)a_{2,t} - g_{2,t} + y_2, \text{ for } t = 1, 2, 3,
\end{align*}
\] (2.3) (2.4)

where \( g_2 \) represents the resources given by a parent to her children, \( a_{2,t} \) denotes the beginning-of-period asset holdings of an age-2 agent at time \( t \), and \( r \) denotes the exogenous rate of return on these assets.

In terms of the strategic behavior of agents in the game played between the parent and the child, I focus on the simplest strategic setting and look at the standard Cournot-Nash equilibrium. I assume that children are Cournot players, that is, they take as given the decisions of their parents when making their own decisions. Hence, I assume away equilibria where children have an active role in the bargaining process. This is a common assumption in the literature and is also the most realistic one when dealing with the relation between parents and underage children: children have no bargaining power, while parents usually make most decisions for their children. O’Connel and Zeldes (1993) show that, given children’s Cournot behavior, the strategic power of parents is irrelevant. That is, if children take as given their parents’ actions, the resulting equilibrium is the same whether parents are Stackelberg leaders, Cournot players that take their children’s decisions as given, or make decisions for their children.

Therefore, the Cournot-Nash equilibrium maximizes the utility of parents. It is therefore not surprising that, when measures of welfare are based on the utility of currently living parents or do not account for the utility of currently living children, departures from the Cournot-Nash equilibrium reduce welfare (e.g., Aiyagari, Greenwood, and Seshadri, 2002).

O’Connel and Zeldes’ (1993) finding also means that when children have no bargaining power we can solve for the equilibrium assuming that all agents are Cournot players. Henceforth, I assume that both parents and children are Cournot players and I consider a Cournot-Nash bargaining model, as is standard in the literature.

The first-order conditions with respect to asset holdings, \( a_{2,t+1} \), and transfers, \( g_{2,t} \), are respec-
tively:

\[ U_c(c_{1,t}) = \beta (1 + r) U_c(c_{2,t+1}) \text{ for } t = 1, 2 \] \hspace{1cm} (2.5)

\[ U_c(c_{2,t}) = \beta_p U_c(c_{1,t}) \text{ for } t = 1, 2 \] \hspace{1cm} (2.6)

The decision function that determines the level of parental transfers for any given level of children’s decision (equation 2.6) and the budget constraints (equations 2.3 and 2.4) can then be used to characterize the relation between the level of parental transfers, parent’s wealth and children’s decisions.

For an utility function that is strictly increasing and strictly concave, we obtain the standard feature of altruistically motivated transfers: any factor that 1) increases parents pre-transfer consumption, or 2) decreases children’s pre-transfer consumption, results in an increase in parental transfers. The marginal utility of transfers for parents is given by the difference between the marginal utility of their own consumption and the marginal utility of their children’s consumption. A decrease in children’s pre-transfer consumption increases the marginal utility of children’s consumption and increases the parent’s marginal utility of transferring resources to her children. Hence, everything else constant, parental transfers increase with parent’s wealth, \( a_{2,t} \), and children’s savings, \( a_{2,t+1} \).

2.2. Welfare maximization

The levels of parental transfers, \( g_{2,t} \), and children’s savings, \( a_{2,t+1} \), that maximize average welfare

\[ W_t = V_{1,t} + V_{2,t}, \] \hspace{1cm} (2.7)

taking as given future decisions\(^3\) are such that:

\[ U_c(c_{1,t})(1 + \beta_p) = U_c(c_{2,t}) \] \hspace{1cm} (2.8a)

\[ U_c(c_{1,t}) = \beta (1 + r) U_c(c_{2,t+1}) \] \hspace{1cm} (2.8b)

\(^3\)This assumption is made for the sake of clarity. The purpose of the optimality conditions is to recollect the rationale presented in Bernheim (1989).
The difference between the optimality condition (2.8a) and the parents’ first order conditions for $g_{2,t}$, equation (2.6), is not trivial for any level of the altruism discount factor, $\beta_p$. Hence, the Cournot-Nash equilibrium is not welfare maximizing, a result that was proved by Bernheim (1989, Theorem 1). The optimality conditions (2.8a) and (2.8b) imply that the level of parental transfers that maximizes current welfare is higher than the one chosen by parents in the Cournot-Nash equilibrium. In the Cournot-Nash equilibrium, parents are indifferent between allocating one extra unit of resources to their own consumption or to their children’s consumption. Children are always better off with a higher level of parental transfers. Therefore, a social planner that weights children’s utility positively also prefers a larger level of parental transfers.

Moreover, if parental transfers are larger, according to the Euler equation, (2.8b), the level of children’s savings is greater. So a social planner would also prefer a larger level of children’s savings.

2.3. Savings and welfare

It is now possible to establish a crucial relationship between the average levels of welfare and a forced increase in savings, which can be interpreted as the result of a borrowing constraint.

From the previous section it is clear that a social planner that seeks to maximize current average welfare would like to implement a higher level of parental transfers. However, in the absence of constraints on gifts and asset accumulation, policies that transfer resources lump-sum across generations are neutral in overlapping generations model with altruistic transfers (see Altig and Davis, 1989). Hence, it is not possible to attain the optimal allocation of resources from parents to children by imposing a lump-sum tax on parents and transferring the revenues to children. But, because when parents make their transfers decisions, they respond to children’s savings, a change on the assets accumulated by children impacts parental transfers and can drive them closer to their socially optimal level.

**Proposition 1:** When the economy is in a Cournot-Nash equilibrium, a marginal increase in children’s savings increases children’s lifetime utility and the average level of welfare.

Proof: The average welfare gains of a marginal increase in children’s savings are given by

$$\frac{\partial W_1}{\partial \theta_{2,2}} = (1 + \beta_p) \left[ -U_c(c_{1,1}) + \beta(1 + r)U_c(c_{2,2}) \right] + U_c(c_{1,1}) \frac{\partial g_{2,1}}{\partial \theta_{2,2}}$$

(2.9)
The first component of this equation describes the welfare impact of a change in children savings through the distortion it introduces on current and future children’s savings decisions, while the second one relates to its impact on the child’s utility through the response it generates in parental transfers.

The welfare gains for the current children and parents of an increase on the savings of current and future children are given, respectively, by

\[
\frac{\partial V_{1,1}}{\partial a_{2,2}} = U_c(c_{1,1}) \frac{\partial g_{2,1}}{\partial a_{2,2}} + \left[ -U_c(c_{1,1}) + \beta (1 + r)U_c(c_{2,2}) \right], 
\]

(2.10)

and

\[
\frac{\partial V_{2,1}}{\partial a_{2,2}} = \beta_p \left[ -U_c(c_{1,1}) + \beta (1 + r)U_c(c_{2,2}) \right].
\]

(2.11)

If the economy is initially at the Cournot-Nash equilibrium, we have

\[
\frac{\partial W_1}{\partial a_{2,2}} = \frac{\partial V_{1,1}}{\partial a_{2,2}} = U_c(c_{1,1}) \frac{\partial g_{2,1}}{\partial a_{2,2}},
\]

(2.12)

and

\[
\frac{\partial V_{2,1}}{\partial a_{2,2}} = 0.
\]

(2.13)

We can use the first-order condition for parental transfers (equation 2.6) and the budget constraints (equations 2.3 and 2.4) to derive the response of parental transfers to a change in children’s savings.

\[
\frac{\partial g_{2,1}}{\partial a_{2,2}} = \frac{\beta_p U_{cc}(c_{1,1})}{U_{cc}(c_{2,1}) + \beta_p U_{cc}(c_{1,1})}
\]

(2.14)

Because \( \beta_p \) is positive and \( U(.) \) is strictly concave, \( 0 < \partial g_{2,1}/\partial a_{2,2} < 1 \), which implies that when children’s savings, \( a_{2,2} \), go up, parental transfers, \( g_{2,1} \), also increase, but by less than children’s savings.

Thus, there is a welfare gain from increasing children’s savings from their Cournot-Nash equilibrium levels and this gain is related to the increase in children’s savings and the response of parental transfers to changes in children’s savings. This increase in savings can correspond to the impact of a small constraint on children’s borrowing. Therefore, Proposition 1 shows that a small borrowing
constraint increases children’s lifetime utility and the average level of welfare.⁴

Notice that when children’s savings are increased in the first period, children’s savings respond optimally in the second period according to:

\[
\frac{\partial a_{2,3}}{\partial a_{2,2}} = \frac{1 + \beta (1 + r)^2 \left[ \frac{U_{cc}(c_{2,3})}{U_{cc}(c_{1,2})} + \beta p \frac{U_{cc}(c_{2,3})}{U_{cc}(c_{2,2})} \right]}{1 + \beta (1 + r)^2} > 0 \quad (2.15)
\]

This relates to the findings in Laitner (1993) where a binding borrowing constraint results in an increase in parental transfers and desired future savings and therefore reduces the likelihood of future binding borrowing constraints.

The asset accumulation decision has two distinct roles in a model with parental altruism. Savings allow children to smooth consumption by reallocating resources across time, and it generates an increase in parental transfers by raising the marginal utility of children’s consumption. So if a child would have any strategic power, she would take advantage of the positive externality she has on her parent’s utility by “working” on her parent’s reaction function. She would choose a higher level of asset accumulation than in the Cournot-Nash equilibrium in order to decrease her current consumption, increase her marginal utility of consumption and thus generate a higher level of parental transfers. By construction, this increase in children’s savings would increase her lifetime utility.

This bargaining perspective leads then to another interpretation of the impact of a forced increase in savings on the outcome of the game played between parents and their children. We can assume that the Cournot-Nash equilibrium occurs because timing is such that children cannot commit to decisions that would place them in a better place in their parents’ reaction function. Children would like to have their parents believe that they would save more; however, this is not credible. A forced increase in savings can then be viewed as a commitment technology that tells parents that children are, in fact, going to save above the unconstrained time-consistent level.

Hence, provided that the transfer motive is active and the offspring take as given the decision of their parents, a forced increase in savings can have a positive impact on their well-being and on the average level of welfare of currently living agents. The assumption of non-strategic behavior by the

⁴If the welfare function is of the more general form \( W_t = \lambda V_{1,t} + (1 - \lambda) V_{2,t} \), with \( \lambda \in (0, 1) \), we get \( \frac{\partial W_t}{\partial a_{2,t}} = \lambda \frac{\partial V_{1,t}}{\partial a_{2,t}} + (1 - \lambda) \frac{\partial V_{2,t}}{\partial a_{2,t}} \) and the results still hold.
underage children is crucial. If they could behave strategically, it is possible that the competitive equilibrium would result in levels of transfers at or above the optimum and an increase in savings would decrease welfare.

These results also hold in a model with two-sided altruism. In the presence of two-sided altruism, assuming that the child discounts the parent’s lifetime utility at the rate $\beta_c$, and the child can transfer a positive amount of resources, $g_{1,t}$, to the parent, the individual optimality conditions for transfers are:

$$g_{1,t} : U_c(c_{1,t}) \geq \beta_c U_c(c_{2,t})$$

$$g_{2,t} : U_c(c_{2,t}) \geq \beta_p U_c(c_{1,t}).$$

While the level of transfers that maximize total welfare (2.7) are such that:

$$(1 + \beta_p)U_c(c_{1,t}) = (1 + \beta_c)U_c(c_{2,t})$$

If $g_{2,t}$ is unconstrained, as is assumed, or if the constraint on $g_{2,t}$ is not binding we have:

$$U_c(c_{1,t}) \geq \beta_c U_c(c_{2,t}), \quad g_{1,t} = 0,$$

$$U_c(c_{2,t}) = \beta_p U_c(c_{1,t})$$

which implies

$$(1 + \beta_p)U_c(c_{1,t}) \geq (1 + \beta_c)U_c(c_{2,t}).$$

This means that even in the presence of two-sided altruism, the level of parental transfers, $g_{2,t}$, that maximizes welfare is higher than the one chosen by parents in the Cournot-Nash equilibrium. Because the impact of savings on welfare depend on the sub-optimality of parental transfers, the results also hold in the case of two-sided altruism.

If parental altruism is of the paternalistic form where rather than caring about the child’s overall happiness parents care about allocations that depend positively on their children’s resources, e.g. children’s consumption, investment on education or human capital accumulation, parental transfers will still be sub-optimal and the results still hold.
However, these results do not hold in a warm-glow model. When parents’ utility is a function of the resources they give to their children, parental transfers are sub-optimal but they do not respond to the change in children’s consumption that results from the increase in savings.

2.4. Savings and welfare in the Long-run

I showed above that the Cournot-Nash equilibrium is not optimal and a forced increase in savings increases the average level of welfare of currently living generations. At this point it is important to examine the long-run implications of increasing children’s savings. In this section, I show that the mechanism described in the previous section generates a higher increase in welfare in the long-run. I then use these findings to rationalize the introduction of human capital accumulation in a model used to evaluate the impact of a borrowing constraint.

**Proposition 2:** When the economy is in a Cournot-Nash equilibrium, a permanent marginal increase in children’s savings increases children’s lifetime utility and the average level of welfare in the first and in the second periods. Moreover, the increase in the average level of welfare is higher in the second period than the in the first one.

**Proof:**

The welfare gains for children of a permanent marginal increase in savings \( \partial a_2 \) are the same as the average welfare gains and, in a Cournot-Nash equilibrium, are given by

\[
\frac{\partial V_{1,t}}{\partial a_2} = U_c(c_{1,t}) \frac{\partial g_{2,t}}{\partial a_2}
\]

(2.20)

both in the long-run, \( t = 2 \), and in the short-run, \( t = 1 \).

We can then use the first-order condition for parental transfers (equation 2.6) to derive the response of parental transfers to a permanent change in children’s savings:

\[
\frac{\partial g_{2,2}}{\partial a_2} = \frac{U_{cc}(c_{2,2})}{U_{cc}(c_{1,2})} \left(1 + r\right) + \beta_p
\]

\[
\frac{U_{cc}(c_{2,2})}{U_{cc}(c_{1,2})} + \beta_p
\]

(2.21)

while

\[
\frac{\partial g_{2,1}}{\partial a_2} = \frac{\beta_p}{\frac{U_{cc}(c_{2,1})}{U_{cc}(c_{1,1})} + \beta_p}
\]

(2.22)
because $\beta_p$ and $r$ are positive and $U$ is strictly concave, $\frac{\partial g_2.2}{\partial a_2}, \frac{\partial g_2.1}{\partial a_2} > 0$. Moreover $\frac{\partial g_{2.2}}{\partial a_2} > 1$ and $1 > \frac{\partial g_{2.1}}{\partial a_2}$. Then, $\frac{\partial g_{2.2}}{\partial a_2} > 1 > \frac{\partial g_{2.1}}{\partial a_2} > 0$. So $\frac{\partial V_{2.2}}{\partial a_2} > \frac{\partial V_{2.1}}{\partial a_2} > 0$. ■

In a three period economy, where agents only accumulate assets for two periods, a permanent increase in savings is equivalent to an increase in savings over the two periods. The impact of the increase in the first period variables can be viewed as its short-run effect, and its impact in the second period variables as its long-run effect.

As current children’s savings increase, future parent’s wealth increases. Parent’s pre-transfer consumption increases which decreases the marginal utility of parent’s consumption and increases her marginal utility of transferring resources to her children. Consequently, wealthier parents transfer more resources to their children. So, a permanent increase in children’s savings leads to a higher increase in future parental transfers than in current ones. Therefore, because the response of parental transfers is higher in the long-run, the impact on the lifetime utility of agents is also higher in the long-run.

A forced increase in savings increases the average level of utility of agents in the short-run and in the long-run with the long-run impact being stronger due to the resulting increase in parents’ wealth.

This simple analysis allows us to understand a consequence of borrowing constraints that has been overlooked in the literature. From Propositions 1 and 2 it is clear that a marginal increase in children’s savings in the standard Cournot-Nash equilibrium increases average welfare both in the short and in the long-run. Therefore a binding borrowing constraint, which results in a forced increase in children’s savings, might also have a positive welfare effect. In an environment where children borrow against future income, a credit constraint, by reducing the amount of borrowing by children, effectively increases children’s savings. The resulting increase in children’s savings reduces children’s consumption and raises their marginal utility of consumption. The optimal response of parents is to increase transfers to children. So the forced increase in children’s savings results in an increase in parental transfers and might move them closer to their socially optimal level.\textsuperscript{5} Hence, a borrowing constraint can move the economy towards the social optimum by inducing an increase in parental transfers and can therefore increase average welfare.

\textsuperscript{5} Cox (1990) presents empirical evidence that parents transfer more resources to their children to alleviate liquidity constraints.
Altig and Davis (1989) have shown that, in a standard overlapping generations model with altruism, borrowing constraints increase welfare in the long-run. However they attribute this effect solely to the long-run general equilibrium effects of the borrowing constraint, namely on the increase in the long-run wage levels due to the rise in capital resulting from the forced increase in savings. Additionally, they presume there are short-run costs of transitioning to this better steady-state which preclude any argument for allowing restrictions on loans.

I have shown that, independently of the pecuniary effects of the increase in savings underlined in Altig and Davis (1989), a small borrowing constraint increases children’s lifetime utility and the average level of welfare in the short and in the long-run.

Assume now that we have a closed infinite horizon version of the economy where savings are channeled to the accumulation of physical capital, $K$. Production depends on capital according to a standard neoclassical production function, $f(K)$. The interest rate is given by

$$r = f'(K) - \delta$$

where $\delta \in (0, 1)$ is the depreciation rate of capital, while individuals’ labor income is given by

$$y_i = \left[f(K) - Kf'(K)\right] s_i \quad i = 1, 2$$

where $s_1 + s_2 = 1$.

The steady-state of the Cournot-Nash equilibrium of the economy is such that:

$$1 + r = \frac{1}{\beta_p \beta}.$$  \hspace{1cm} (2.23)

If $\beta_p \beta < 1$, capital is below the golden rule level of capital and we can say that the Cournot-Nash equilibrium results in under accumulation of capital in the sense that it is possible to generate higher levels of consumption and welfare in a steady-state with more capital. Once we allow for pecuniary effects, the increase in labor income resulting from the rise in capital, due to the increase in savings, generates extra welfare gains as shown in proposition 3 in Altig and Davis (1989). As a consequence, the pecuniary effects emphasized in Altig and Davis (1989) can further improve the long-run impact of a binding borrowing constraint on welfare by bringing the aggregate level of
capital closer to its golden rule level. However, it can also decrease long-run welfare if it pushes the aggregate level of capital above that level.

Thus, the question concerning the impact of a borrowing constraint is whether it places the economy closer to the optimal path, enhancing average welfare, or further away beyond it, decreasing average welfare. Since I cannot characterize analytically the steady-states and the transition paths from an equilibrium where children borrow against future income, after the introduction of a borrowing constraint, I study numerically the impact of borrowing constraints in key economic variables and welfare.

In addition, it is usually presumed that borrowing constraints lead to underinvestment in human capital. De Gregorio (1996) and Christous (2001) have shown that by reducing human capital accumulation, borrowing constraints have negative effects on the level of human capital and on growth. If I allow for the endogenous accumulation of human capital in the three period economy, a forced increase in savings has a negative impact on human capital. If $h_{2,t+1} = H(e_t)$, where $e_t$ is the amount of physical resources allocated by children to the accumulation of human capital in period $t$ and $H(.)$ is a strictly increasing and strictly concave function, then $\frac{dW_1}{d\alpha_2}$ and $\frac{dg_2}{d\alpha_2}$ are positive at the Nash-Cournot equilibrium. But $\frac{de_1}{d\alpha_2} < 0$, which implies that human capital decreases when children’s savings rise.

Therefore, the presence of human capital accumulation might diminish or even reverse, the long-run effects that were discussed earlier. On the one hand, future parents will be less wealthy and might transfer fewer resources to their children. On the other hand, the decrease in human capital reduces worker’s productivity offsetting the pecuniary effects of the increase in physical capital.

Hence, in the next section, to look at the impact of a borrowing constraint on welfare, I construct a more realistic economic environment that allows for the endogenous accumulation of human capital and where prices are flexible.

3. An extended economy with altruism

I study an economy where a large number of identical agents are born in each period and live for $T$ periods, first as children and then as adults. Individuals in each generation maximize their
discounted lifetime utility. For someone born in period $t$ this is given by

$$V_{1,t} = U(c_{1,t}, l_{1,t}) + \beta V_{2,t+1}$$

while for older agents we have

$$V_{j,t} = \sum_{i=j}^{T} \beta^{i-j} U(c_{i,t+i-j}, l_{i,t+i-j}) + \beta_p f V_{j-1,t} \quad j = 2, \ldots, T$$

where $\beta > 0$ is the intertemporal discount factor, $c_{i,t}$ is consumption, and $l_{i,t}$ is leisure of an age $- i$ individual in period $t$. Agents are assumed to have $f$ children in the second period of their lives. A parent values her children’s consumption and leisure because she cares for their well-being. $\beta_p \in [0, 1/(\beta f))$ is the discount factor for their offspring’s lifetime utility. Furthermore, children have the same preferences as adults over their own consumption and leisure.

The “momentary” utility function is assumed to take the constant relative risk aversion form of a Cobb-Douglas consumption-leisure index,

$$U(c, l) = \frac{(c^\sigma l^{1-\sigma})^{1-\rho}}{1-\rho},$$

where $\rho$ is the coefficient of risk aversion, and $\sigma$ is the coefficient of consumption on the Cobb-Douglas index.

The exogenous fertility rate of the population is $f$, so that a younger generation is $f$ times bigger than the preceding one. The share of age $- i$ individuals in the population, given by the measure $\mu_i$, $i = 1, 2, \ldots, T$, is constant over time, and $\mu_{i+1} = \frac{1}{f} \mu_i$, with $\sum_{i=1}^{T} \mu_i = 1$.

Individuals have one unit of time each period to allocate to work, education, and leisure. In the first period of their lives, agents can choose how much time they allocate to leisure, education, and work. Before their mandatory retirement they can work for $(T - 1)$ periods supplying $h_{i,t}$ hours of labor and earning $w_t h_{i,t} s_{i,t}$, where $w_t$ and $s_{i,t}$ are the real hourly wage rate per unit of human capital and age $- i$ agent’s level of human capital in period $t$, respectively. In the last period of their lives they retire and consume or bequeath the value of their assets.

Agents in this economy accumulate claims on real capital used in production by firms. The
budget constraint facing an individual of age \( i \) at time \( t \) can be written as

\[
a_{i+1,t+1} = (1 + r_t)a_{i,t} - g_{i,t} + g_{i+1,t}/f + w_t h_{i,t} s_{i,t} - c_{i,t} - e_{i,t},
\]

(3.4)

where \( a_{i,t} \) denotes the beginning-of-period asset holdings of an \( age - i \) individual at time \( t \), and \( r_t \) denotes the rate of return on these assets. The variable \( e_{i,t} \) describes private investment in education. Finally, \( g_{i,t} \) represents the resources (in terms of the consumption good) given by a parent to her children, so \( g_{i+1,t}/f \) are the resources received by \( age - i \) agent from her \( age - (i+1) \) parent. Without loss of generality, I allow these transfers from parents to occur twice during their lifetime: in the second period of parents’ lives when their offspring are children and in the last period of parents’ lives.

I assume that agents are Cournot players in the interaction with their parents. This is, equivalent to assuming that children take as given the resources they receive from their parents. They simply receive whatever transfer is given, and they cannot manipulate their parent’s decision. A more realistic assumption would be to allow adult parents to make decisions in behalf of their underage children, but as noted in section (2), both assumptions result in a Nash-Cournot equilibrium. The determinant factor is that children do not have any bargaining power.\(^6\)

Transfers from \( age - T \) parents to their offspring, \( g_{T,t} \), cannot be negative, but I allow inter-vivo transfers from \( age - 2 \) parents to their children, \( g_{2,t}, \) to be negative. That is, \( age - 2 \) parents can make children transfer resources to them.\(^7\) Henceforth, I will refer to these two types of transfers distinctively as bequests and parental transfers respectively.

I study two economies. In the first one, the unconstrained economy, children can borrow against future income. In the second one, the borrowing constrained economy, children cannot borrow

\(^6\)The assumption that underage children do not act strategically in the interaction with their parents is crucial but is also realistic. However, I allow adult children to overlap several periods with their parents, and strategic behavior can then emerge between them. If children behave strategically, the equilibrium might result in levels of transfers at or above the optimum and an increase in savings would then reduce welfare. Whether, given the nature of the game played by underage children and their parents, the strategic behavior of adult children in the relation with their parents would change the results significantly depends on whether the outcome of the early interactions is reversed and the lifetime transfers from parents to their offspring is no longer sub-optimal. This is a question that, to my knowledge, has not been answered in the literature and will also not be addressed in this paper. In effect, I abstract from strategic behavior to focus on the role of the channels associated with the relation between adult parents and their underage children.

\(^7\)This assumption means that age-2 parents can use their children’s resources, acquired through borrowing or child labor, for instance, to finance their own consumption. Note however that in the equilibria of the calibrated version of the model we only observe positive inter-vivo transfers.
against their future income:

\[ a_{2,t} \geq 0, \forall t. \]  \hfill (3.5)

Children accumulate human capital by going to school. The level of human capital accumulated by each child increases with the time allocated to learning, \( d_{1,t} \), and the quality of the education service. The quality of the service provided is assumed to be an increasing function of the total level of physical resources invested, \( e_{1,t} \).

This education process is represented by the following technology:\(^8\)

\[ s_{2,t+1} = \theta e_{1,t}^{\eta_e} d_{1,t}^{\eta_d} \]  \hfill (3.6)

where the parameters \( \eta_d \) and \( \eta_e \) are respectively the coefficients of time and physical resources in the learning technology while \( \theta \) is the total factor productivity of the education process.

The production technology of the economy is described by a constant-returns-to-scale function,

\[ Y_t = K_t^{1-\alpha} L_t^\alpha, \]  \hfill (3.7)

where \( \alpha \in (0, 1) \) is the labor share of output, and \( Y_t \), \( K_t \), and \( L_t \) are the levels of output, capital input, and effective labor input, respectively.

The capital stock is equal to the aggregate asset holdings of individuals in the economy. It depreciates at a constant rate \( \delta \) and evolves according to the law of motion,

\[ K_{t+1} = (1 - \delta) K_t + I_t. \]  \hfill (3.8)

The effective labor input is given by the number of hours worked by agents in the economy weighted by their levels of human capital,

\[ L_t = N_t \sum_{i=1}^{T-1} \mu_{i,t} s_{i,t} h_{i,t}, \]  \hfill (3.9)

where \( N_t \) is the size of the population in period \( t \).

Competitive firms maximize profits, equal to \( Y_t - \delta K_t - w_t L_t - r_t K_t \), taking the wage, \( w_t \),

\(^8\)This learning technology is similar to the one in Glomm and Ravikumar (1992) and Soares (2005) for instance.
and the interest rate, $r_t$, as given. The first-order conditions for the firm’s problem determine the following functions for the net real return to capital and the real wage rate:

$$r_t = (1 - \alpha)(\frac{K_t}{L_t})^{-\alpha} - \delta,$$

$$w_t = \alpha(\frac{K_t}{L_t})^{1-\alpha}.$$

(3.10)

3.1. Optimality and social welfare

I evaluate equilibria using utilitarian social welfare functions that, as noted by Samuelson (1968), should be used to analyze the normative aspects of economic policy. Pareto optimality is not a persuasive criterion as it is too lenient in evaluating equilibria, supporting a wide range of equilibria where agents fare in very distinct ways, and is too strict in evaluating changes to the economic environment. This is particularly true in overlapping generations economies, where it is rarely possible to make any of the currently living agents better off without making at least another one worse off. As in Diamond (1965), Samuelson (1968), Atkinson and Sandmo (1980), Ghiglino and Tvede (2000), and Erosa and Gervais (2000) among others, a utilitarian social welfare function can be the discounted sum of successive generations’ lifetime utility:

$$SW_t = \sum_{i=1}^{T} \mu_i V_{i,t} + \sum_{j=1}^{\infty} (\beta_p f)^j \mu_1 V_{1,t+j}$$

(3.11)

where $\beta_p \in [0, 1/f]$ is the social discount factor, the rate at which the central planner discounts the utility of future generations. The “social planner” maximizes the weighted well-being of agents living in the economy, currently and in the future, taking into account the well-being stemming from altruism. But, due to altruism, the lifetime utility of future generations is counted multiple times, and this social planner’s function not only is time-inconsistent, but it biases the evaluation of policies towards the ones that generate higher gains in the long-run.\(^9\) Moreover, economic theory provides no guidance for the choice of the weight, $\beta_p f$, given to the welfare of future generations. As such the choice of a social welfare function introduces a significant amount of subjectivity in the analysis. To deal with this issue, I do not aggregate welfare across periods, so I do not have to take a stand on the relative importance of future generations. I concentrate on evaluating how

\(^9\)See Bernheim (1989) for a thorough discussion of this problem.
changes to the economic environment affect the distribution of welfare across agents and on the average lifetime utility of agents's living in each period:

\[ W_t = \sum_{i=1}^{T} \mu_i V_{i,t}. \]  

(3.12)

4. Calibration

To solve the model numerically I assign values to the parameters of preferences and technologies. I calibrate the steady-state of the closed economy where children cannot borrow against future income. I assume that agents in this model live for 5 periods and the model period is 17 years long. Agents are born at the age of 1 becoming adult workers at age 17, they then can work for 51 years and retire thereafter to a total real-life age of 85 years.

Fertility Rate

The exogenous fertility rate is calibrated to match the observed population growth rate for the US economy in the last decades, 0.012 (Citibase Data, 1946-1993). For the five generation model, this translates to a growth rate of \( f = 1.2248 \).

Preferences

I set the coefficient of risk aversion \( \rho \) equal to the standard value, 2, and choose the values for the discount factor, \( \beta \), and the coefficient of consumption in the utility function, \( \sigma \), so that steady-state capital-output is approximately 3.32 and, on average, agents in the labor force allocate a third of their time to labor (see Cooley and Prescott (1995)). I set the coefficient of consumption in the utility function, \( \sigma \), equal to 0.29 and I set \( \beta \) to be 0.69.

Altruism

I calibrate the altruistic discount factor, \( \beta_p \), to 0.54 to match the average ratio of spending on public primary and secondary education to aggregate expenditures on consumption in the US economy, 0.053, as in Fernandez and Rogerson (1995).

Production Technology

The share of labor in the production function is set to 0.6 following Cooley and Prescott (1995). The annual depreciation rate is 6.4%, so that the steady-state annual investment/capital ratio is

\[ \text{The value obtained for } \beta_p \text{ \ is very similar to the one obtained by Nishiyama (2000, 2002) which calibrated this parameter to match the relative size of intergenerational transfers.} \]
0.076.

**Education Technology**

The evidence is mixed on the magnitude of the impact of school quality on learning with a very wide range of estimates for the elasticity of the increase in educational attainment with respect to spending per pupil. Card and Krueger (1996) survey the literature and find that estimates of the elasticity of earnings with respect to spending per pupil in 25 studies range from 0.01 to 0.29, with the average of the estimates being 0.16. According to Betts (1996), studies that use a functional form for the education production function similar to the one in this paper tends to generate higher estimates for the elasticities. I therefore calibrate the coefficient of expenditures on education in the education production function to 0.2, as in Fernandez and Rogerson (1995).

The coefficient corresponding to the time dedicated to education is chosen in order to match the average percentage of available time dedicated to education. Juster and Stafford (1991) find that school aged children allocate about 29.41% of their time to school work. Like total factor productivity in the goods technology, total factor productivity in the education technology only has a scale effect on most variables; it does not affect the time allocations or factor prices and impacts on all other variables by a factor of $\theta^{1-\theta c}$. For computational reasons, I set its value to 10.

The parameter choices for the benchmark model are summarized in table 1.

5. **Findings**

I first shut down the general equilibrium effects of the borrowing constraint and look at a partial equilibrium where I maintain constant factor prices. This allows me to analyze the impact of the borrowing constraint on individuals’ decisions and on welfare while abstracting from its pecuniary externalities. For this purpose, I set the wage and interest rate to their equilibrium levels in the steady-state of the unconstrained economy. I then take into account the pecuniary effect of the borrowing constraint on individuals’ welfare by looking at the general equilibrium where factor prices are endogenous.
5.1. Steady-State

I present the steady-state results in Table 2. In the first column, I summarize the results for the unconstrained economy. Although, they receive a significant amount of resources from their parents, it is optimal for children to finance consumption and expenditures on education by also borrowing against their future income.

In the second column, I show the partial equilibrium results for the economy where children cannot borrow against future income. This equilibrium can be viewed as the equilibrium path of a single family or as the equilibrium in a small open economy that takes as given international factor prices. In the third column, I present the general equilibrium results for the borrowing constrained economy.

I measure the welfare benefit of an agent in the economy with borrowing constraints as the fixed percentage increase in the lifetime consumption of an individual of the same age and her descendents in the steady-state of the economy without borrowing constraints needed to equate the level of welfare of both individuals. I refer to this measure as the compensating variation. The compensating variation is positive (negative) if there is a welfare gain (loss) relatively to the steady-state without borrowing constraints.

In this example, although, given the level of parental transfers, children want to borrow to consume and invest in education, agents are better off in the steady-state when children are not allowed to borrow. Consumption would have to increase by about 3.5% for a newly born agent to be as well off in the steady-state without the borrowing constraint as in the steady-state with the borrowing constraint.

The imposed increase in children’s asset accumulation resulting from the introduction of the borrowing constraint reduces the resources available to children for education and consumption, $w_1s_{1,t}h_{1,t} + \frac{a_{2,t}}{f} - a_{2,t+1}$. But the level of transfers they receive from parents is higher than in the initial equilibrium.\footnote{As noted by Altig and Davis (1989), in the presence of borrowing constraints, all parental transfers occur in the period where children are constrained. In the unconstrained economy, the timing of transfers is irrelevant and, for simplification, I assume they occur in the first period.}

In partial equilibrium, the constraint generates a decrease in children’s consumption and investment in education as the increase in parental transfer is lower than the increase in children’s savings.
and hence is not strong enough to compensate for the impossibility of borrowing. This contrasts with the outcome from the simpler model in section 2 (see Proposition 2) and results in part from the persistency of the impact of the borrowing constraint on the level of skills and, hence, on the income level of parents. It also underlines the importance of accounting for the human capital accumulation process.

Furthermore, as the level of skills decreases, the effective real wage rate, $w_t s_{i,t}$, decreases, and agents substitute consumption for leisure in response to the decline in the relative cost of leisure in terms of consumption. As labor supply and the skill level decreases, income also decreases and, although agents do not have to repay any debt, they further reduce consumption. But, the reduction in consumption is offset by the increase in leisure, and the steady-state utility levels are higher than in the unconstrained equilibrium.

In general equilibrium, the increase in aggregate savings resulting from the borrowing constraint generates a rise in capital accompanied by an increase in wages, a decrease in interest rates, and a surge in output. In the long-run, the rise in parental transfers is more than enough to compensate for the impossibility of borrowing, and both consumption and investment in education increase. Because there is a significant increase in wealth, leisure also increases for most generations, and the rise in welfare is much higher than when factor prices remain constant. Consumption would have to increase by about 8.5% for a newly born agent to be as well off in the steady-state without the borrowing constraint as in the steady-state with the borrowing constraint. Notice, that the level of human capital is lower than in the initial equilibrium as, although they invest more physical resources in education, children reduce the amount of time they allocate to education to enjoy more leisure.

From these experiments it is clear that both the response of parental transfers and the change in factor prices discussed in Altig and Davis (1989) play an important role in generating a significant welfare impact of the borrowing constraint. In general equilibrium, both channels feed off each other, which makes it difficult to distinguish their contributions to the observed welfare gains. In order to have a better idea of the relative impact of those two channels, I shut-down the parental transfer channel and compute the general equilibrium steady-state of the constrained economy where I maintain parental transfers fixed at their level in the unconstrained economy. As can be seen in the last column of Table 2, the increase in the wage rate is much more significant in this
environment, but welfare increases much less than in the other equilibria. Consumption would have to increase by only about 1.7% for a newly born agent to be as well off in the steady-state without the borrowing constraint as in the steady-state with the borrowing constraint. Hence the contribution of parental transfers to the observed welfare gains in terms of compensating variation is somewhere between 3.5% and 6.8%. These numbers underline the important role of the parental transfer channel in generating the welfare gains associated with the introduction of the borrowing constraint.

Although there are long-run benefits from imposing a borrowing constraint in the benchmark economy, everything else constant children would like to borrow against future income to finance consumption and investment in human capital. When faced with a borrowing constraint, young agents respond by investing less in education and they will have less human capital. Hence, there might exist short-run costs, not accounted for in our simpler model, that can preclude any gains from allowing for this constraint. To analyze the short-run welfare impact of the borrowing constraint, I look at the transition from the steady-state in the unconstrained economy to the steady-state of the constrained one.

5.2. Transition path

I first show the partial equilibrium transition where I maintain the factor prices constant. I then look at the general equilibrium transition where factor prices are endogenous.

5.2.1. Partial equilibrium

The central role played by the “altruism” mechanism described in section 2 is clear as the resources transferred by parents to their offspring increase very significantly when the borrowing constraint becomes effective. This factor is crucial and has been overlooked in the literature. In Figure 7.1, we observe that in the first period, when the borrowing constraint is introduced, the transfers from parents increase significantly (panel d); parents of current children (age-2 agents) increase inter-vivo transfers, while older parents increase the amounts they will bequeath to their offspring in the last period of their lives. In the graph we see bequests increasing and peaking for the progenitors of current age-2 parents. Older parents increase bequests which period after period trickle down to current age-2 agents. This increase in bequests takes place in order to diminish the burden on the
younger generations of parents who can use those future bequests to finance current consumption and parental transfers.

As a consequence of individuals’s responses to the introduction of the borrowing constraint, there is an initial surge in aggregate savings resulting mostly from the reduction in debt, while the aggregate level of human capital decreases significantly in the period following the onset of the borrowing constraint as can be seen in Figure 7.3.

Most of the additional parental transfers are allocated by children to consumption which decreases by about 4% (panel a), while the resources allocated to education immediately decrease significantly more, by almost 16%, (panels e and f), and then increase slightly to their new steady-state values, remaining drastically below their initial levels. But as we can see in Figure 7.2 panel a, children’s lifetime utility increases from the onset of the borrowing constrained equilibrium. This increase in welfare is related to the surge in leisure (Figure 7.1 panel b) as the initial period’s children not only allocate much less time to education but they also work less when older. These findings support the claim from Section 2 that the borrowing constraint can make children better off by placing them in a better location in the reaction function of their parents.

Notice also that in the period where the borrowing constraint takes effect, the welfare of age-2 parents increases (see Figure 7.2 panel a). Facing a higher level of asset accumulation from their children, parents find it optimal to increase the resources they transfer to them. In order to do so they increase the time allocated to work (see Figure 7.1 panel c) and reduce current and future consumption (see Figure 7.1 panel a). Hence, although their own older parents also increase the amount they will bequeath them, for the initial period parents, the increase in parental transfers comes at the cost of lower consumption and leisure levels for the remaining of their lives. This implies that, in this economy, the “selfish” utility of the initial parents, that is utility levels derived only from own consumption and leisure, decreases (see Figure 7.2 panel b). However, the significant rise in children’s lifetime utility\footnote{Note that the increase in children’s welfare is a result of the increase in their "selfish" lifetime utility and in the altruistic component of their lifetime utility which depends on future children’s utility.} increases parents’ lifetime utility’s altruistic component more that offsetting the previous effect and in the initial period’s age-2 parents' lifetime utility increases. In this calibration, only the lifetime utility of the oldest generation decreases in the short-run. More importantly, in Figure 7.2, it is clear that the average lifetime utility of agents increases immediately
in response to the borrowing constraint.\footnote{In other calibrations of the model economy the results are qualitatively similar, but in some cases parents’ lifetime utility decreases slightly. Nonetheless, children’s and the average lifetime utilities increase in all periods in all cases studied.}

For the reasons presented in section 3.1, I chose to look at the average level of utility of the agents living in each period. Nevertheless, because both average welfare and the lifetime utility of children are higher in every period after the introduction of the borrowing constraint, we can infer that any measure of welfare of the type described by (3.11) is also higher than in the unconstrained equilibrium. Hence the borrowing constraint generates an increase in welfare for any welfare function of the form:

\[
SW_t = \sum_{i=1}^{T} \mu_i V_{i,t} + \sum_{j=1}^{\infty} (\beta p f)^j \mu_1 V_{1,t+j}
\] (5.1)

For the current calibration, these utility gains come at the cost of the first period age-5 agents.

In addition, we observe, in figure 7.2 panel b, that children’s as well as the average levels of “selfish” lifetime utility increase following the introduction of the borrowing constraint. Hence, if we construct a welfare measure that ignores the altruistic components of agents’ utilities we also obtain a welfare improvement with the introduction of the constraint on children’s ability to borrow.

5.2.2. General equilibrium

The difference between the general equilibrium and the partial equilibrium paths stems from the adjustment of factor prices and its feedback into agents’ decisions. As can be seen in Figure 7.4, in general equilibrium, the initial increase in savings observed in the partial equilibrium path (see figure 7.3) is spread out over time and sustained leading to a higher level of physical capital in the long-run. As physical capital increases, the wage rate rises, and the interest rate decreases. The response of the wage rate implies a rise in the return to education and in parents’ labor income generating funds that are channeled to education. There is a rise in investment in education relatively to the partial equilibrium path. As a result, initially, human capital drops much less than in partial equilibrium and converges to considerably higher levels afterward although still below its
As income increases, consumption also rises and the levels of welfare are higher than in partial equilibrium. Not only is the immediate increase in the average level of welfare higher in general equilibrium (see figures 7.6 panel a and 7.2 panel a), but children are much better off with the introduction of the borrowing constraint while older generations are slightly worse off in the initial periods because of the initial decrease in wages and subsequent decrease in interest rates. Afterwards, welfare increases steadily to a higher long-run level. By looking at Figures 7.2 panel b and 7.6 panel b we can see that most of the difference in the short-run levels of welfare is due to the increase in the “selfish” lifetime utility of children.

Therefore the pecuniary effects of the increase in savings underlined in Altig and Davis (1989) are important as they augment the positive impact of the borrowing constraint on welfare in the long-run by bringing the aggregate level of capital closer to its golden rule level. However, they are not essential as, even in the absence of any general equilibrium effects, a borrowing constraint increases average welfare in the short and in the long-run.

6. Final comments

This paper looks at the impact of a constraint on children’s borrowing on welfare. The presumption that such constraints are an obstacle to efficient levels of education serves to rationalize political designs that attempt to replicate the unconstrained equilibrium. But, in an overlapping generations economy with altruism, the unconstrained Nash-Cournot equilibrium leads to sub-optimal levels of parental transfers and does not maximize the welfare of children nor the average welfare of currently living agents. I show that by forcing an increase in children’s savings that induces an increase in parental transfers, a borrowing constraint can lead to an increase in children’s welfare as well as in the average levels of welfare both in the short-run and in the long-run. In a calibrated model where children invest in human capital, the welfare impact of a borrowing constraint can be quite large. So, agents can be better off in an economy with incomplete credit markets, and setting public policy instruments in order to replicate the complete markets equilibrium might not be welfare improving.

\footnote{For other calibrations of the model, in general equilibrium the level of human capital rebounds to levels above the initial ones. While the amount of physical resources allocated to education increases to levels above the initial one in all the other examples, the levels of human capital also depend on the amount of time allocated to education which decreases in this calibration.}
Although we might not want to replicate the allocation of resources obtained in the unconstrained equilibrium, the credit constrained equilibrium is at most a second best. However, it is well known that if parents are altruistic and we have interior solutions for savings and private transfers, policies that transfer resources across generations are neutral, as any public transfers are exactly offset by a change in private parental transfers. But, Altig and Davis (1989) show that, even in the presence of altruism and an active transfer motive, policies that transfer resources across generations can have a real effect in a borrowing constrained economy. So, the presence of a credit constraint can also have a positive welfare impact by enlarging the set of functional policies as intergenerational transfer policies become effective in the constrained economy and increase welfare. Hence, even when its direct impact on welfare is negligible a borrowing constraint can help improve the welfare of agents in the economy by allowing for a wider set of policy instruments.

In this paper, I do not look at the impact of restrictions on the ability of parents to borrow against future income. Because in practice, parents are also widely affected by credit constraints, the impact of constraints on parents’ ability to borrow is an issue that is worth considering in environments where agents are altruistically linked to older parents. As I noted, depending on the type of interaction between middle-aged agents and their parents, it is possible that a borrowing constraint that affects adult’s decisions has a similar effect to the one underlined in the paper, increasing inter-vivo transfers from older to middle-aged agents. Thus, if we are in a Cournot-Nash equilibrium, a borrowing constraint that restricts parents’ borrowing can also be welfare improving.

Finally, credit constraints seem to be closely related to child labor, and this paper shows that a credit constraint that does not allow children to borrow might actually benefit children. It would therefore be important to analyze how child labor responds to this type of borrowing constraint and what does it imply for children’s well-being in an environment where they allocate time to labor as well as the acquisition of human capital. Additionally, it would also be interesting to see how the imposition of a ban on child labor affects welfare as it might trigger a response similar to the one at the core of the results in this paper. In fact, a binding ban on child labor effectively eliminates a source of income, reducing children’s consumption levels and increasing their marginal utility of consumption. Parents respond by increasing their transfers to children. A ban on child labor can therefore make children better off by generating an increase in parental transfers.

Hence, this paper presents an important albeit simple channel through which the economic
environment affects the well-being of children; a channel that can be useful in analyzing policies aimed at improving children's welfare.

References


### 7. Tables and Figures

#### Table 1 - Calibration

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#### Table 2 - Steady-State Equilibria

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Figure 7.1: Individual variables along partial equilibrium path.
Figure 7.2: Welfare Levels along Partial Equilibrium Path.
Figure 7.3: Aggregate variables along partial equilibrium path.
Figure 7.4: Aggregate variables along general equilibrium path.
Figure 7.5: Individual variables along general equilibrium path.
Figure 7.6: Welfare Levels along General Equilibrium Path.