HYDROSTATIC FORCES ON SURFACES

Remember the second law of Pascal –

*In a container, pressure acts perpendicular to the boundary*

In this lecture we will investigate how forces act on surfaces –

- Nature of plane or curved surface
- Total force
- Center of force

Remember – $F = P \times A$
Chapter 4  Forces Due to Static Fluids

a) Fluid power cylinder

b) Storage tank

c) Fluid reservoir and hatch

d) Tank with a curved surface

e) Retaining wall

f) Aquarium observation windows

FIGURE 4.1  Examples of cases where forces on submerged areas must be computed.
Example Problem 4.2:

What is the pressure and force at the bottom of the containers??

\[ P = 0 + \gamma_o \times 2.4 + \gamma_w \times 1.5 \]

\[ \gamma_o = 9.81 \times 0.90 = 8.83 \text{ kN/m}^3 \]

\[ P = 0 + 8.83 \times 2.4 + 9.81 \times 1.5 \]

\[ P = 0 + 21.2 + 14.7 = 35.9 \text{ kPa (gage)} \]

\[ F = PA = 35.9A \]
\[ = 35.9 \times \pi (3.0)^2/4 \]
\[ = 253.8 \text{ kN} \]
Force and pressure for container 2 will be the same! – Pascal’s paradox!

Force and pressure are being felt by the insides of the container.

However the weight of the two containers will be different

W_1 = \pi \frac{(3.0)^2}{4} \times [2.4 \times 8.83 + 9.81 \times 1.5] \\
= 253.8 \text{ kN}

Container 2

Volume of frustum of cone = 

V = \frac{\pi}{12} \times h \times (D_2^2 + D_d + d^2)

D_1 = 1.2 \\
D_2 = 2.307 \\
D_3 = 3.0 \\

W_2 = V_1 \times 8.83 + V_2 \times 9.81 \\
= \frac{\pi}{12} \times 2.4 \times (2.307^2 + 2.307 \times 1.2 + 1.2^2) \times 8.83 + \frac{\pi}{12} \times 1.5 \times (3^2 + 3 \times 2.307 + 2.307^2) \\
= 134.7 \text{ kN}
Forces on rectangular walls

**FIGURE 4.6**  Vertical rectangular wall.

Hoover dam! Water depth ~ 720 ft. Pressure at bottom ??????
**Problem 4.4**

Determine the force and center of force on a WALL if -

Fluid = gasoline = sg = 0.68

Total depth = 12 ft
Length of wall = 40 ft

**Average pressure** = $\gamma \ast \frac{h}{2}$

$= 0.68 \ast 62.4 \ast \frac{12}{2} = 254.6$

**Force on wall** = $F = p \ast A$

$= 254.6 \ast 12 \ast 40 = 122,204$ lb

**Center of pressure** = $12/3$ from bottom

$= 4$ ft from bottom
**Problem 4.5**

Dam retaining water.
Dam length = 30.5 m
Depth = 8m
Dam wall inclined at 60 degrees
Calculate force and location of force?

Average pressure $= \gamma \cdot h/2$

$= 9.81 \cdot 8/2 = 39.24$

Length along which pressure acts =

$= L = h/\sin \theta$
= 8 / sin 60°

= 9.24 m

**Force** = pA = 39.24 * 9.24 * 30.5

= 11060 kN

**Center of pressure** =

h/3 from bottom = 8/3 = 2.67 m

or

L/3 along inclined plane = 9.24/3 = 3.08 m
Force on a submerged plane area

Find the force on the gate and the center of pressure.

Steps –

1. find the centroid of the area
2. find the distance from the top of fluids to centroid = hc
3. determine the pressure at the centroid
4. determine the force at the centroid
5. calculate the moment of inertia of the area
6. compute the center of pressure
centroid of given rectangle = 1.2/2 = 0.6 m

hc = 3.0 + 0.6 = 3.6 m

**pressure** = p = γ * hc

= p = 9.81*3.6 = \textbf{35.32 kN/m}^2

**Force** = pA

= 35.32 * 1.2 * 2 = \textbf{84.7 kN}

Now,

Moment of inertia of rectangle = Ic = BH^3/12

= 2 * 1.2^3/12 = 0.288

**The center of pressure** =

= hc + Ic/(hc*A)

= 3.6 + 0.288/(3.6*1.2*2) = \textbf{3.633 m}
7. Force on a submerged plane area

Problem 4.6:
**STEPS –**

1. determine the point where the angle of inclination intersects the fluid surface
2. locate the centroid of the surface
3. determine \( h_c \) – vertical distance from fluid surface to centroid
4. determine \( L_c \) – inclined distance to centroid
   \[ h_c = L_c \sin \theta \]
5. calculate area \( A \)
6. calculate force on area = \( F_R = \gamma h_c A \)
7. calculate the moment of inertia = \( I_c \)
8. calculate the center of pressure = \( L_p = L_c + I_c / (L_c A) \)
9. Sketch \( F_R \) acting on the area
10. \( h_p = L_p \sin \theta \)
11. or \( h_p = h_c + I_c \sin^2 \theta / (h_c A) \)
GIVEN –
Tank with oil, \( \text{sg} = 0.91 \)

Rectangular gate \( B = 4 \text{ft}, H = 2 \text{ft} \)

Inclined wall of tank = \( \theta = 60 \) degrees

Centroid of the gate is = \( hc = 5 \text{ft} \) from the surface

Calculate –

- The force on the gate
- The center of pressure
1. **draw a sketch** of the system

2. **identify the centroid of the gate** based on its geometry

3. \( h_c = 5 \text{ft} \)

4. **determine \( L_c \)**

   \[
   \frac{h_c}{L_c} = \sin \theta
   \]

   \[
   L_c = \frac{h_c}{\sin \theta} = \frac{5}{\sin 60^\circ} = 5.77 \text{ ft}
   \]

5. **area of gate** = \( A = BH = 4 \times 2 = 8 \text{ft}^2 \)

6. **Determine force on the gate**

   \[
   F_r = \gamma_o h_c A = 0.91 \times 62.4 \times 5 \times 8 = 2270 \text{ lb}
   \]

7. **Determine center of pressure**

   moment of inertia of gate = \( I_c = BH^3/12 = 4 \times 2^3 / 12 = 2.67 \text{ft}^4 \)

   **Center of pressure** = \( L_p = L_c + (I_c/L_c*A) \)

   \[
   = 5.77 + (2.67/5.77 \times 8) = 5.77 + 0.058 = 5.828 \text{ ft}
   \]
Example
Centroid at ????????????

**Force** = \( \gamma h_c \, A = 62.4 \times (6+1) \times 0.5 \times 2 \times 3 \)

= 1310 lb

**Center of pressure** = \( h_p = h_c + \frac{I_c}{hcA} \)

= 7 + \left[ \frac{2 \times 3^3}{36 / (7 \times 0.5 \times 2 \times 3)} \right] = 7.07 \text{ ft}
Assignment # 3:

- 4.9E
- 4.11M
- 4.17M
- 4.20M
**Distribution of Force on a Curved surface**

First step – isolate the portion of interest and visualize the forces
Problem 4.8:

$h_1 = 3.00 \text{ m}$

$h_2 = 4.5 \text{ m}$

$w = 2.5 \text{ m}$

liquid = water

Compute the **horizontal and vertical components on the curved surface** and the resultant force.

1. the isolated volume is shown below –
Determine the volume above the curved surface

Areas –

Area = A1 + A2 = (3 * 1.5) + \frac{1}{4} (\pi * 1.5^2)
\[ 4.5 \text{ m}^2 + 1.767 \text{ m}^2 = 6.267 \text{ m}^2 \]

Volume = area x width = \(6.267 \times 2.5 = 15.67 \text{ m}^3\)

Therefore weight = \(F_v = 15.67 \times 9.81 = 153.7 \text{ kN}\)

**Determine the centroid of the vertical force**

For A1, \(x_1 = 0.75\)

For A2, \(x_2 = 0.424 \text{ R} \text{ (formula for centroid of quadrant)}\)

\[ = 0.424 \times 1.5 = 0.636 \text{ m} \]

Determine the centroid of the area A1 and A2 =

\[ x = \frac{(A_1x_1 + A_2x_2)}{(A_1 + A_2)} = 0.718 \text{ m} \]

**Now let’s compute the horizontal force**

Depth to the centroid of projected area

\[ h_c = h1 + s/2 = 3.0 + 0.75 = 3.75 \text{ m} \]

**Magnitude of the horizontal force =**

\[ F_h = \gamma h_c * sw = 9.81 * 3.75 * 1.5 * 2.5 = 138.0 \text{ kN} \]
Depth to the line of action of the horizontal force

\[ hp = h_c + \frac{s^2}{12} \]
\[ h_c = 3.75 + \frac{1.5^2}{12 \times 3.75} = 3.80 \text{ m} \]

Resultant Force \( Fr = (F_v^2 + F_h^2)^{1/2} \)

\[ = (153.7^2 + 138^2)^{1/2} \]
\[ = 206.5 \text{ kN} \]

Angle of inclination of the resultant force \( = \tan^{-1} \left( \frac{F_v}{F_h} \right) \)

\[ = \tan^{-1} \left( \frac{153.7}{138.0} \right) \]
\[ = 48.1^\circ \]
Assignment # 4:

• 4.47M