Learning Long Distance Phonotactics

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I present a learner which learns the attested long distance phonotactic patterns in the world’s languages.

This learner:
(1) keeps track of the order of sounds—but not the distance between them (precedence relations)
(2) fails to learn logically possible—but unattested—long distance phonotactics

The conclusion is if humans generalize in the way suggested by the model, it can explain features of the typology of long distance phonotactics (cf. Moreton 2008, *analytic bias*)
1 Introduction
   - Long Distance Phonotactics
   - Representing Long Distance Phonotactics

2 Precedence-based Learning
   - Learning in Phonology
   - Precedence Grammars

3 Conclusion
   - Issues
   - Summary
1. **Introduction**
   - Long Distance Phonotactics
   - Representing Long Distance Phonotactics

2. **Precedence-based Learning**
   - Learning in Phonology
   - Precedence Grammars

3. **Conclusion**
   - Issues
   - Summary
What is long distance phonotactics (LDP)?

Long Distance Agreement (LDA) patterns are those within which particular segments, separated by at least one other segment, must (dis)agree in some feature (Hansson 2001, Rose and Walker 2004).

- Hansson (2001) adds that the intervening segments are not audibly affected by the agreeing feature.
- This is in order to clearly distinguish LDA from spreading (see also Gafos 1999 and Walker 1998).
In well-formed words, sibilants agree in the feature [anterior].

1. \([s, z, ts, ts', dz]\) never precedes \([ʃ, ʒ, tʃ, tʃ', dʒ]\).
2. \([ʃ, ʒ, tʃ, tʃ', dʒ]\) never precedes \([s, z, ts, ts', dz]\).

Examples (Sapir and Hojier 1967):

1. \(ʃiːteːʒ\) ‘we (dual) are lying’
2. \(daʃdoːliʃ\) ‘he (4th) has his foot raised’
3. \(*ʃiːteːz\) (hypothetical)
4. \(*daʃdoːliʃ\) (hypothetical)
In well-formed words, sibilants agree in the feature [anterior], but only the [-anterior] sibilants are ‘active’.

1. \([s,z,ts,dz]\) never precedes \([ʃ,ʒ,ʃ,ʒ]\].

Examples (Hansson 2001, citing Cook 1979,1984):

1. \(ʃɪʔʃíʔd\dot{a}\)? ‘my duck’
2. \(nāʃyāʃ\) ‘I killed them again’
3. \(*zítʃíʔd\dot{a}\)? (hypothetical)
4. \(*s\dot{nāʃyāʃ}\) (hypothetical)
Examples of long distance phonotactics

- Consonantal Harmony (Hansson 2001, Rose and Walker 2004)
  - Sibilant, liquid, dorsal, voicing, … harmony and disharmony
  - Symmetric/Asymmetric LDA
  - ~120 languages documented with consonantal harmony (Hansson 2001).

- possibly Vowel Harmony with ‘transparent’ vowels
  - Finnish, Hungarian, Nez Perce (see Baković 2000 and references therein)
  - Some controversy over how transparent: see Gordon (1999), Gafos and Benus (2003), and Gick et. al. (2006).
One debate, two puzzles

Debate: Is it really non-local?

Puzzles
- How do we explain the absence of blocking in the typology?
- (if it non-local) How such non-local patterns learned?
Debate: Is LDP really spreading?

- Spreading means the intervening segments are affected.
- Nasal spreading in Malay (Johore dialect, Walker 1999, citing Onn 1980)

1. mõnõwõn ‘to capture’ (active)
2. põŋwâsan ‘supervision’

Navajo as spreading (+/- indicates [anterior])

3. fi:te:3 ‘we (dual) are lying’
4. da:sdo:lis ‘he (4th) has his foot raised’

- Gafos (1999) argues that Navajo=Navajo’ (see Hansson 2001 for counterarguments).
Spreading means the intervening segments are affected.

Nasal spreading in Malay (Johore dialect, Walker 1999, citing Onn 1980)

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Gafos (1999) argues that Navajo=Navajo’ (see Hansson 2001 for counterarguments).
Puzzle # 1: Explaining the typology of LDP

- The typology of LDA is notable in two respects (Hansson 2001, Rose and Walker 2004):
  1. LDA holds between similar segments.
  2. Blocking patterns are absent.
- The latter helps distinguish LDA from spreading.
In well formed words, voiceless sibilants agree in the feature [anterior] unless, between two voiceless sibilants which disagree in [anterior], there is a voiced sibilant (and no other voiceless sibilants).

1. [ʃ] never precedes [s] unless, for each [ʃ], a [z] or [ʒ] occurs between [ʃ] and its nearest following [s]
2. [s] never precedes [ʃ] unless, for each [s], a [z] or [ʒ] occurs between [s] and its nearest following [ʃ]

Examples (all hypothetical since no language example exists!):

The absence of this type of LDP is robust!

Consenus has formed in the few proposed counterexamples (Sanskrit, Kinyarwanda) that they are better analyzed as spreading (Schein and Steriade 1986, Mpiranya and Walker 2005).
Rose and Walker (2004) take both gaps as systematic.

Their Agreement By Correspondence (ABC) analysis of LDA in OT uses:

- **CC-Correspondance constraints**: two consonants are in correspondence if they are sufficiently similar (agnostic about similarity metric)
- **ID-CC(FEATURE)** constraints which enforce agreement of FEATURE for corresponding consonants.

This is intended to capture both the similarity and blocking effects.
Hansson (2007) studies the predicted typology of ABC and shows the ABC approach *does* predict non-local blocking effects of certain types. ...reluctantly suggests that the absence of blocking patterns is accidental.
But it fails... hence Puzzle #1

- Hansson (2007) studies the predicted typology of ABC and shows the ABC approach *does* predict non-local blocking effects of certain types.
- ...reluctantly suggests that the absence of blocking patterns is accidental.

Current theory doesn’t explain the absence of blocking in the typology of LDP
Arbitrarily many segments may intervene between agree-ers.

- Albright and Hayes (2003a) observe that “the number of logically possible environments... rises exponentially with the length of the string.”
- Thus there are potentially too many environments for a learner to consider in discovering LDP patterns.
However, does “arbitrarily many” really require a learner to consider every logically possible nonlocal environment?
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The possible words of English can be thought of a set which includes:

\{ slam, fist, blick, flump, \ldots \}

and which excludes:

\{ sram, fizt, bnick, flumk, \ldots \}
Phonotactics as sets

The binary, categorical distinction between ‘well-formed’ and ‘ill-formed’ is a convenient abstraction.

kip > 0wi:ks > bzəɾʃk

What kind of sets are long distance phonotactic sets?

<table>
<thead>
<tr>
<th>word</th>
<th>Navajo</th>
<th>Sarcee</th>
<th>Hypothetical</th>
</tr>
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<td>✓</td>
<td>✓</td>
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<td>...</td>
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</tbody>
</table>
What kind of sets are long distance phonotactics?

- **Long distance phonotactic patterns are regular.**
  

- Regular sets have many characterizations (see e.g. Kracht 2003). They are those sets describable with:
  
  - finite state acceptors
  - right-branching rewrite grammars
  - regular expressions
  - monadic second order logic
FSAs

(1) can be related to finite state OT and rule-based models, which allow us to compute a phonotactic finite-state acceptor (Johnson 1972, Kaplan and Kay 1994, Karttunnen 1998, Riggle 2004), which becomes the target grammar for the learner.

(2) are well-defined and can be manipulated.

(Hopcroft et. al. 2001).
1. \([s,z,ts,ts’,dz]\) never precedes \([ʃ,ʒ,ʃʃ,ʃʃ’,dʒ]\).
2. \([ʃ,ʒ,ʃʃ,ʃʃ’,dʒ]\) never precedes \([s,z,ts,ts’,dz]\).

**Symmetric LDP: Navajo**

C = any consonant except sibilants
s = [+anterior] sibilants
V = any vowel
ʃ = [-anterior] sibilants

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<tr>
<td>ʃos</td>
<td>ʃos</td>
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<tr>
<td>sots</td>
<td>ʃtos</td>
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<tr>
<td>ʃʃʃʃ</td>
<td>…</td>
</tr>
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<td>…</td>
<td>…</td>
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</tbody>
</table>
This grammar recognizes an infinite number of legal words, just like the generative grammars of earlier researchers.

It does accept words like [tnʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃʃ pinMode-1
1. \([s, z, ts, dz]\) never precedes \([ʃ, ʒ, tʃ, dʒ]\).

- \(C\) = any consonant except sibilants
- \(s\) = [+anterior] sibilants
- \(V\) = any vowel
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<table>
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<tr>
<td>ʃots</td>
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<td>ʃoʃos</td>
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```
C = any consonant except sibilants
s = [+anterior] voiceless sibilants
V = any vowel
ʃ = [-anterior] voiceless sibilants
z = any voiced sibilant

Accepts | Rejects
---|---
sos | sofʃ
ʃoʃ | ʃos
ʃotozotos | ʃtozosʃos
... | ...
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Learning in Optimality Theory
Tessier(2006)]

Learning in Principles and Parameters

Learning Phonological Rules

Learning Phonotactics
Hayes and Wilson(2008), Goldsmith and Riggle(submitted)]
What is Learner so that Language of G’ = Language of G?

See Nowak et. al. (2002) and Niyogi (2006) for overviews.
Learning cannot take place unless the hypothesis space is restricted. 

G’ is not drawn from an unrestricted set of possible grammars.

The hypotheses available to the learner ultimately determine:

1. the kinds of generalizations made
2. the range of possible natural language patterns

Under this perspective, Universal Grammar (UG) is the set of available hypotheses.
Different Kinds of Hypothesis Spaces are Learned Differently.

- The set of syntactic hypotheses available to children is not the same as the set of phonological hypotheses available to children.
  - The two domains do not have the same kind of patterns and so we expect them to have different kinds of learners.

- Likewise, the set of LDP patterns are different from patterns which restrict the distribution of adjacent, contiguous segments.
Different kinds of phonotactic constraints can be learned by different learning algorithms.

A complete phonotactic learner is a combination of these different learning algorithms.

Here, I am only showing how one part of the whole learner—the part that learns long-distance constraints—can work.
Some concerns regarding identification in the limit from positive data

- No noise in input
- No requirement for learner to be efficient
- No requirement on ‘small’ sample to succeed
- Exact identification is too strict a criterion
What learner can acquire the machines above from finite samples of Navajo, Sarcee, respectively?

This question is not easy. There is no simple ‘fix’.

The class of regular sets is known to be insufficiently restrictive for learning to occur!
(Gold 1967, Osherson et. al. 1986, Jain et. al. 1999).
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(Gold 1967, Osherson et. al. 1986, Jain et. al. 1999).
Some subclasses of the regular languages are sufficiently restrictive for learning to occur

- Locally $k$-testable in the strict sense (Strictly Local)
- Locally $k$-testable
- Many others from grammatical inference community
Locally 2-testable in the strict sense (Strictly Local)

- \( sotos \in L \iff \{so, ot, to, os\} \subseteq G_L \)
- E.g. bigrams
Locally 2-testable in the strict sense (Strictly Local)

- $sotos \in L$ iff $\{so, ot, to, os\} \subseteq G_L$
- E.g. bigrams

Locally 2-testable

- $sotos \in L$ iff $\{so, ot, to, os\} \in G_L$
- E.g sets of bigrams
The Sub-regular Hierarchy
(McNaughton and Papert 1971, Pullum and Rogers 2007)

Locally 2-testable in the strict sense (Strictly Local)

- $sotos \in L \iff \{so, ot, to, os\} \subseteq G_L$
- E.g. bigrams

Locally 2-testable

- $sotos \in L \iff \{so, ot, to, os\} \in G_L$
- E.g sets of bigrams

Noncounting

- there is some $n > 0$, for all $uv^n w \in L \iff uv^{n+1} w \in L$ for all strings $u, v, w \in \Sigma^*$.
- E.g. closure of Locally Testable class under concatenation and boolean operations.
Spreading is locally 2-testable in the strict sense
Symmetric LDP is locally 1-testable
Asymmetric LDP and Hypothetical are noncounting
The Sub-regular Hierarchy
(McNaughton and Papert 1971, Pullum and Rogers 2007)

- Spreading
- Symmetric LDP
- Asymmetric LDP
- LDP with Blocking

- Strictly Local
- Locally Testable
- Non-Counting (Locally Testable w/ order)
- Regular

The goal!
Outline

1 Introduction
   • Long Distance Phonotactics
   • Representing Long Distance Phonotactics

2 Precedence-based Learning
   • Learning in Phonology
     • Precedence Grammars

3 Conclusion
   • Issues
   • Summary
Order matters, but not distance.

- Shawe-Taylor and Christianini (2005, chap. 11) also discuss kernels defined over discontiguous, ordered strings for use in text classification.
1. \([s,z,ts,ts',dz]\) never precedes \([ʃ,ʒ,ʃʃ,ʃʃ',dʒ]\).  
2. \([ʃ,ʒ,ʃʃ,ʃʃ',dʒ]\) never precedes \([s,z,ts,ts',dz]\).
1. \([s,z,ts,ts',dz]\) never precedes \([ʃ,ʒ,tʃ,tʃ',dʒ]\).
2. \([ʃ,ʒ,tʃ,tʃ',dʒ]\) never precedes \([s,z,ts,ts',dz]\).

\[=\]

\[\begin{align*}
[s] \text{ can be preceded by } [s]. \\
[s] \text{ can be preceded by } [t]. \\
... \\
[t] \text{ can be preceded by } [s]. \\
... \\
[ʃ] \text{ can be preceded by } [ʃ]. \\
[ʃ] \text{ can be preceded by } [t]. \\
... 
\end{align*}\]
A precedence grammar is a list of the allowable precedence relations in a language.
Words recognized by a precedence grammar are those for which every precedence relation is in the grammar.

Example. (Assume $\Sigma = \{s, f, t, o\}$.)

Precedence $G = \{ (s, s) \quad (s, t) \quad (s, o) \\
                 (f, f) \quad (f, t) \quad (f, o) \\
                 (t, s) \quad (t, f) \quad (t, t) \quad (t, o) \\
                 (o, s) \quad (o, f) \quad (o, t) \quad (o, o) \}$. 

(1) The Language of $G$ includes $sotos$. 
Languages Recognized by Precedence Grammars

- Words recognized by a precedence grammar are those for which every precedence relation is in the grammar.

- Example. (Assume $\Sigma = \{s,f,t,o\}$.)

  $\text{Precedence } G = \begin{cases} 
  (s,s) & (s,t) & (s,o) \\
  (f,f) & (f,t) & (f,o) \\
  (t,s) & (t,f) & (t,o) \\
  (o,s) & (o,f) & (o,t) & (o,o) 
\end{cases}.$

(1) The Language of $G$ includes $sotos$. 
Words recognized by a precedence grammar are those for which every precedence relation is in the grammar.

Example. (Assume $\Sigma = \{s, f, t, o\}$.)

$$
\text{Precedence } G = \left\{ (s,s), (s,t), (s,o), (f,f), (f,t), (f,o), (t,s), (t,f), (t,t), (t,o), (o,s), (o,f), (o,t), (o,o) \right\}.
$$

(1) The Language of $G$ includes $sotos$. 
Words recognized by a precedence grammar are those for which every precedence relation is in the grammar.

Example. (Assume $\Sigma = \{s,f,t,o\}$.)

$$
\text{Precedence } G = \left\{ 
\begin{array}{ccc}
(s,s) & (s,t) & (s,o) \\
(f,f) & (f,t) & (f,o) \\
(t,s) & (t,f) & (t,o) \\
o,s) & (o,f) & (o,t) & (o,o)
\end{array} \right\}.
$$

(1) The Language of $G$ includes *sotos*.
Words recognized by a precedence grammar are those for which every precedence relation is in the grammar.

Example. (Assume $\Sigma = \{s,f,t,o\}$.)

\[
\text{Precedence } G = \begin{cases} 
(s,s) & (s,t) & (s,o) \\
(f,f) & (f,t) & (f,o) \\
(t,s) & (t,f) & (t,o) \\
o,s) & (o,f) & (o,t) & (o,o) 
\end{cases}.
\]

(1) The Language of $G$ includes $sotos$. 

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Words recognized by a precedence grammar are those for which every precedence relation is in the grammar.

Example. (Assume $\Sigma = \{s, f, t, o\}$.)

Precedence $G = \left\{ \begin{array}{llll}
(s, s) & (s, t) & (s, o) \\
(f, f) & (f, t) & (f, o) \\
t, s & t, f & t, t & (t, o) \\
o, s & o, f & (o, t) & (o, o) \\
\end{array} \right\}$. 

(1) The Language of $G$ includes $sotos$. 

Words recognized by a precedence grammar are those for which every precedence relation is in the grammar.

Example. (Assume $\Sigma = \{s, f, t, o\}$.)

\[
\text{Precedence } G = \left\{ \begin{array}{ccc}
(s,s) & (s,t) & (s,o) \\
(f,f) & (f,t) & (f,o) \\
(t,s) & (t,f) & (t,o) \\
(o,s) & (o,f) & (o,t) & (o,o)
\end{array} \right\}.
\]

(1) The Language of $G$ includes sotos.
(2) The Language of $G$ excludes sotof.
Words recognized by a precedence grammar are those for which every precedence relation is in the grammar.

Example. (Assume $\Sigma = \{s, j, t, o\}$.)

Precedence $G = \{(s, s) \ x \ (s, t) \ (s, o) \ (j, j) \ (j, t) \ (j, o) \ (t, s) \ (t, j) \ (t, t) \ (t, o) \ (o, s) \ (o, j) \ (o, t) \ (o, o)\}.$

(1) The Language of $G$ includes $sotos$.
(2) The Language of $G$ excludes $sotoj$. 
Precedence Languages are Regular.

These grammars are notational variants.

Symmetric LDP (e.g. Navajo)

See Heinz (2007) on how to write a finite-state acceptor given a precedence grammar.
Navajo Fragment. (Assume $\Sigma = \{s, f, t, o\}$.)

2. $[f]$ never precedes $[s]$.

\[
\text{Precedence } G = \begin{cases} 
(s, s) & (s, t) & (s, o) \\
(f, f) & (f, t) & (f, o) \\
t, s) & (t, f) & (t, t) & (t, o) \\
o, s) & (o, f) & (o, t) & (o, o) 
\end{cases}.
\]

- The learner has already generalized; it accepts $[fot]$, $[fotot]$, $[sototos]$.
- but not words like $[ftos]$ or $[sosof]$.
Navajo Fragment. (Assume $\Sigma = \{s,f,t,o\}$.)

2. $[f]$ never precedes $[s]$.

Learning

Precedence $G = \{ \}$.

Sample = { }

- The learner has already generalized; it accepts $[fof]$, $[ftot]$, $[sototos]$
- but not words like $[ftos]$ or $[sosof]$
Navajo Fragment. (Assume $\Sigma = \{s, f, t, o\}$.)

2. $[f]$ never precedes $[s]$.

Learning Precedence $G = \{(s, s), (s, o), (t, s), (t, o), (o, s), (o, o)\}$.

Sample = { tosos }

- The learner has already generalized; it accepts $[fof]$, $[fot]$, $[sotos]$
- but not words like $[ftos]$ or $[sosof]$
Learning Precedence Grammars: Navajo Fragment

Navajo Fragment. (Assume $\Sigma = \{s, j, t, o\}$.)

1. [s] never precedes [j].
2. [j] never precedes [s].

$$\text{Learning Precedence } G = \left\{ \begin{array}{ccc}
(s,s) & (s,o) \\
(j,\allowbreak j) & (j,t) & (j,o) \\
(t,s) & (t,\allowbreak j) & (t,o) \\
(o,s) & (o,\allowbreak j) & (o,t) & (o,o) \\
\end{array} \right\}.$$ 

Sample = { tosos, jotoj } 

- The learner has already generalized; it accepts [jof], [jot], [sototos]
- but not words like [jtos] or [sosoj]
Navajo Fragment. (Assume $\Sigma = \{s,f,t,o\}$.)

2. $[f]$ never precedes $[s]$.

Learning Precedence $G = \{(s,s), (s,t), (s,o), (f,f), (f,t), (f,o), (t,s), (t,f), (t,t), (t,o), (o,s), (o,f), (o,t), (o,o)\}$. 

Sample = \{ tosos, ftof, stot \}

- The learner has already generalized; it accepts $[fot]$, $[fot]$, $[sotos]$ 
- but not words like $[ftos]$ or $[sosof]$
Learning Precedence Grammars: Navajo Fragment

Navajo Fragment. (Assume $\Sigma = \{s, f, t, o\}$.)

1. [s] never precedes [f].
2. [f] never precedes [s].

Learning Precedence $G =$

\[
\begin{pmatrix}
(s,s) & (s,t) & (s,o) \\
(f,f) & (f,t) & (f,o) \\
(t,s) & (t,f) & (t,o) \\
(o,s) & (o,f) & (o,t) & (o,o)
\end{pmatrix}
\]

Sample = { tosos, /oto/, stot }

- The learner has already generalized; it accepts [fot], [ftot], [sototos]
- but not words like [ftos] or [sosof]
Navajo Fragment. (Assume $\Sigma = \{s, f, t, o\}$.)

1. [s] never precedes [f].
2. [f] never precedes [s].

Learning Precedence $G = \begin{cases} (s,s) & (s,t) & (s,o) \\ (f,f) & (f,t) & (f,o) \\ (t,s) & (t,f) & (t,o) \\ (o,s) & (o,f) & (o,t) & (o,o) \end{cases}$.

Sample = { tosos, fofot, stot }

- The learner has already generalized; it accepts [fot], [fotot], [sototos]
- but not words like [fotos] or [sosoʃ]
Sarcee Fragment. (Assume $\Sigma = \{s, t, o\}$.)

1. $[s]$ never precedes $[t]$.

Precedence $G = \{(s,s), (s,t), (s,o), (l,s), (l,l), (l,t), (t,s), (t,l), (t,t), (t,o), (o,s), (o,l), (o,t), (o,o)\}$.

- The learner has already generalized; it accepts $[lolt], [lott], [sototos], [lototos]$.
- but not words like $[soso]$
Learning Precedence Grammars: Sarcee Fragment

Sarcee Fragment. (Assume $\Sigma = \{s, f, t, o\}$.)

1. $[s]$ never precedes $[f].$

Learning Precedence $G = \{\}$. 

Sample = \{ \}

- The learner has already generalized; it accepts $[fof], [fot], [sototos], [fotos]$.
- but not words like $[sosof]$
Sarcee Fragment. (Assume $\Sigma = \{s, f, t, o\}$.)


Learning Precedence $G =$ \[
\begin{pmatrix}
(s, s) & (s, o) \\
(t, s) & (t, o) \\
(o, s) & (o, o)
\end{pmatrix}.
\]

Sample = \{ tosos \}

- The learner has already generalized; it accepts $[fof]$, $[ftot]$, $[sotos]$, $[fotos]$
- but not words like $[sosof]$
Learning Precedence Grammars: Sarcee Fragment

Sarcee Fragment. (Assume $\Sigma = \{s, t, o\}$.)

1. $[s]$ never precedes $[t]$.

Learning Precedence $G = \begin{cases}
(s,s) & (s,o) \\
(f,f) & (f,t) & (f,o) \\
(t,s) & (t,f) & (t,o) \\
(o,s) & (o,f) & (o,t) & (o,o)
\end{cases}$.

Sample = { tosos, foton }
Sarcee Fragment. (Assume $\Sigma = \{s,f,t,o\}$.)

1. [s] never precedes [f].

Learning Precedence $G = \{(s,s), (s,t), (s,o), (f,s), (f,f), (f,t), (f,o), (t,s), (t,f), (t,t), (t,o), (o,s), (o,f), (o,t), (o,o)\}$.

Sample = \{ tosos , fotof , fots \}

- The learner has already generalized; it accepts [fot], [fot], [sototos], [fotos]
- but not words like [soso]
Sarcee Fragment. (Assume $\Sigma = \{s,f,t,o\}$.)


Learning Precedence Grammar $G = \{(s,s), (s,t), (s,o), (f,s), (f,f), (f,t), (f,o), (t,s), (t,f), (t,t), (t,o), (o,s), (o,f), (o,t), (o,o)\}$.

Sample = \{ tosos, fotof, fotts \}

- The learner has already generalized; it accepts $[fos]$, $[tot]$, $[sototos]$, $[fotos]$
- but not words like $[soso]$
Learning Precedence Grammars: Sarcee Fragment

Sarcee Fragment. (Assume $\Sigma = \{s, f, t, o\}$.)


Learning Precedence $G = \{(s,s), (s,t), (s,o), (f,s), (f,f), (f,t), (f,o), (t,s), (t,f), (t,t), (t,o), (o,s), (o,f), (o,t), (o,o)\}$.

Sample = { tosos, ftofs, ftots }  

- The learner has already generalized; it accepts [fofs], [ftot], [sototos], [fotos]  
- but not words like [soso$]
Any symmetric or asymmetric LDP pattern (e.g. Navajo and Sarcee) can be described with a precedence grammar.

Any symmetric or asymmetric LDP pattern can be learned efficiently in the manner described above.
The number of logically possible nonlocal environments increases exponentially with the length of the word.

- Precedence-based learners do not consider every logically possible nonlocal environment. They cannot learn logically possible nonlocal patterns like:
  
  (1) If the third segment after a sibilant is a sibilant, they must agree in [anterior].
  
  (2) If the second, third, or fifth segments after a sibilant is a sibilant, they must agree in [anterior].
  
  (3) and so on
Locality and LDP

- Precedence-based learners do not distinguish on the basis of distance at all.
- In one sense, every segment is adjacent to every preceding segment.
- The notion of “arbitrarily many may intervene”—not being able to count distance, while keeping track of order—is sufficiently restrictive for learning to occur.
Precedence-based learners do not distinguish on the basis of distance at all.

In one sense, every segment is adjacent to every preceding segment.

The notion of “arbitrarily many may intervene”—not being able to count distance, while keeping track of order—is sufficiently restrictive for learning to occur.
Hypothetical Fragment. (Assume $\Sigma = \{s, f, t, o, z\}$.)

1. $[f]$ never precedes $[s]$ unless, for each $[f]$, a $[z]$ or $[\bar{z}]$ occurs between $[f]$ and its nearest following $[s]$.
2. $[s]$ never precedes $[f]$ unless, for each $[s]$, a $[z]$ or $[\bar{z}]$ occurs between $[s]$ and its nearest following $[f]$.

Learning Precedence $G =$ \[
\begin{cases}
& \\
\end{cases}
\] .

Sample $= \{ \}$

- The learner has failed to learn Hypothetical! E.g. it accepts $[fos]$. 
The Precedence Learner cannot learn LDP with blocking

Hypothetical Fragment. (Assume $\Sigma = \{s, j, t, o, z\}$.)

1. $[j]$ never precedes $[s]$ unless, for each $[j]$, a $[z]$ or $[3]$ occurs between $[j]$ and its nearest following $[s]$
2. $[s]$ never precedes $[j]$ unless, for each $[s]$, a $[z]$ or $[3]$ occurs between $[s]$ and its nearest following $[j]$

Learning Precedence $G = \begin{pmatrix}
(s,s) & (s,o) \\
(t,s) & (t,o) \\
(o,s) & (o,o)
\end{pmatrix}$.

Sample = $\{\text{tosos}\}$

The learner has failed to learn Hypothetical! E.g. it accepts $[jos]$. 
The Precedence Learner cannot learn LDP with blocking

Hypothetical Fragment. (Assume $\Sigma = \{s, f, t, o, z\}$.)

1. $[f]$ never precedes $[s]$ unless, for each $[f]$, a $[z]$ or $[\overline{z}]$ occurs between $[f]$ and its nearest following $[s]$
2. $[s]$ never precedes $[f]$ unless, for each $[s]$, a $[z]$ or $[\overline{z}]$ occurs between $[s]$ and its nearest following $[f]$

Learning Precedence $G = \left\{ \begin{array}{ccc} (s, s) & (f, f) & (s, o) \\ (f, t) & (f, o) & \\ (t, s) & (t, f) & (t, o) \\ (o, s) & (o, f) & (o, t) \\ & (o, o) & \end{array} \right\}.$

Sample = \{ tosos, fotof \}

- The learner has failed to learn Hypothetical! E.g. it accepts $[f]os$. 

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The Precedence Learner cannot learn LDP with blocking

Hypothetical Fragment. (Assume $\Sigma = \{s, f, t, o, z\}$.)

1. $[f]$ never precedes $[s]$ unless, for each $[f]$, a $[z]$ or $[z]$ occurs between $[f]$ and its nearest following $[s]$
2. $[s]$ never precedes $[f]$ unless, for each $[s]$, a $[z]$ or $[z]$ occurs between $[s]$ and its nearest following $[f]$

Learning Precedence $G = \begin{align*} 
&\{(s,s) \quad (s,t) \quad (s,o) \quad (f,s) \quad (f,f) \quad (f,t) \quad (f,o) \quad (f,z) \\
&(t,s) \quad (t,f) \quad (t,t) \quad (t,o) \\
&(o,s) \quad (o,f) \quad (o,t) \quad (o,o) \quad (o,z) \\
&(z,s) \quad (z,o) \end{align*}$. 

Sample = $\{tosos, fotof, fozos\}$

The learner has failed to learn Hypothetical! E.g. it accepts $[fos]$. 
The Precedence Learner cannot learn LDP with blocking

Hypothetical Fragment. (Assume $\Sigma = \{s, f, t, o, z\}$.)

1. $[f]$ never precedes $[s]$ unless, for each $[f]$, a $[z]$ or $[\bar{z}]$ occurs between $[f]$ and its nearest following $[s]$.

2. $[s]$ never precedes $[f]$ unless, for each $[s]$, a $[z]$ or $[\bar{z}]$ occurs between $[s]$ and its nearest following $[f]$.

Learning Precedence $G =$

$$
\begin{align*}
(s,s) & (s,t) & (s,o) \\
(f,s) & (f,f) & (f,t) & (f,o) & (f,z) \\
t,s) & (t,f) & (t,t) & (t,o) \\
o,s) & (o,f) & (o,t) & (o,o) & (o,z) \\
z,s) & & (z,o)
\end{align*}
$$

Sample $= \{ \text{tosos}, \text{fotof}, \text{fozos} \}$

The learner has failed to learn Hypothetical! E.g. it accepts $[jos]$. 
The Precedence Learner cannot learn LDP with blocking

Hypothetical Fragment. (Assume $\Sigma = \{s, f, t, o, z\}$.)

1. $[f]$ never precedes $[s]$ unless, for each $[f]$, a $[z]$ or $[z]$ occurs between $[f]$ and its nearest following $[s]$.

2. $[s]$ never precedes $[f]$ unless, for each $[s]$, a $[z]$ or $[z]$ occurs between $[s]$ and its nearest following $[f]$.

Learning Precedence $G = \{(s,s), (s,t), (s,o), (f,s), (f,f), (f,t), (f,o), (f,z), (t,s), (t,f), (t,t), (t,o), (o,s), (o,f), (o,t), (o,o), (o,z), (z,s), (z,o)\}$.

Sample = \{ tosos , ftof , fozos \}

- The learner has failed to learn Hypothetical! E.g. it accepts $[f]os$. 

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If humans generalize in the way suggested by the precedence learner, it explains why

(1) there are long-distance phonotactic patterns
(2) there are no long-distance phonotactic with blocking patterns
If humans generalize in the way suggested by the precedence learner, it explains why

1. there are long-distance phonotactic patterns
2. there are no long-distance phonotactic with blocking patterns
1 Introduction
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Why not just use \( n \)-grams over tiers?


- E.g. vowel tiers, consonant tiers, sibilant tiers

\[
\begin{array}{c}
\text{ʃ} \\
\uparrow \\
\text{ʃ} \\
\text{iː} \\
\text{t} \\
\text{eː} \\
\uparrow \\
\text{ʒ}
\end{array}
\quad \text{‘we (dual) are lying’}
\]
Consider a word from Hypothetical.

```
  s  z  ŋ
↑   ↑  ↑
 s  o  t  o  z  o  ŋ (hypothetical)
```

- Maybe only project voiceless sibilants in this case?
- What is the theory of tiers? Cf.
  - Rose and Walker’s agnosticism about what is appropriate similarity metric
  - Hayes and Wilson’s antecedently given tiers
- but see also Goldsmith and Xanthos (2006)
Phonotactic patterns are gradient; this is categorical.

- Nothing in the design on the model depends on its categorical nature.
- There are many ways to make the model gradient:
  - minimum distance length (Ellison 1994), Bayes law (Tenenbaum 1999, Goldwater 2006), maximum entropy (Goldwater and Johnson 2003, Hayes and Wilson 2008), kernel methods (Shawe-Taylor and Christianini 2005), and approaches inspired by Darwinian-like processes (Clark 1992, Yang 2000)
Nothing in the design on the model depends on its categorical nature.

<table>
<thead>
<tr>
<th>precedes</th>
<th>s</th>
<th>ß</th>
<th>t</th>
<th>o</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>0.01</td>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ß</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>o</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Compute cells by calculating the joint probability over precedence relations
- Compute cells by calculating conditional probability of a segment (given all preceding segments)
- Evaluate utility of precedence model with MDL (Goldsmith and Riggle, under review)
Can Precedence Learning occur in the presence of noise?

a. What if certain precedence relations are not in the sample?
b. What if there are just a few exceptions to the constraint?

Angluin and Laird (1988) show that there are classes of languages which, under certain noisy conditions, which can be “probably approximately correctly” learned (Valiant 1984, Kearns and Vazirani 1994).

Precedence languages are such a class.

It remains to be seen exactly what the precedence learner which handles noise looks like.
(4) Precedence Learning can learn ‘unmotivated’ LDP patterns. E.g. “[b] never precedes [3].”

- What do people do?
- Independently motivated restrictions can be built into this grammar to further restrict the hypothesis space.
  - Similarity restrictions on potential agree-ers (Hansson 2001, Rose and Walker 2004) (See also Frisch et. al. 2004)
  - Relevency Conditions on interveners (Jensen 1974) (See also Odden 1994).
- Use the independently motivated theory of similarity to set Bayesian priors over the precedence-based hypothesis space
This independence is a good thing

- Other models require independently motivated theory of similarity (OT-CC, tiers)
- Here, such a theory is not needed for learning
- Allows us to study these factors independently
- What is the contribution of sound similarity to learning phonological patterns?
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A learner which keeps track of order—and not distance—(i.e. precedence relations) learns attested long distance phonotactics, and explains a key feature of the typology—absence of blocking. This helps explain why LDP is distinct from spreading.

We ought to investigate

- How successful as grammars w.r.t MDL
- How to integrate similarity
- Whether predictions are confirmed by language acquisition studies
Acknowledgements

Thank You.

In well formed words:

1. [ʃ] is never preceded by [s].
2. [s] is never preceded by [ʃ] unless the nearest preceding [ʃ] is immediately followed by [n,t,l].

Examples (Applegate 1972, Poser 1982):

1. ksunonus ‘I obey him’
2. kʃunotʃ ‘I am obedient’
3. *ksunonuʃ (hypothetical)
4. kʃunots (hypothetical)
5. ʃtíjepus ‘he tells him’
6. *sustimeʃ (hypothetical)
7. ʃʃlusiʃin ‘they (dual) are gone awry’
1. \([\text{s}]\) is never preceded by \([\text{s}]\).
2. \([\text{s}]\) is never preceded by \([\text{s}]\) unless the nearest preceding \([\text{s}]\) is immediately followed by \([\text{n}, \text{t}, \text{l}]\).

Precedence Grammars as given cannot describe the pattern in Chumash.

*\(k\)inots (hypothetical)
\(tijepus\) ‘he tells him’

Next I will show how to extend precedence grammars to capture patterns like those found in Chumash.

- Bigram Precedence
- Relative Precedence
The grammar contains elements of the form \((ab,c)\):
“\([c]\) can be preceded by \([ab]\)”.

The idea is that in Chumash
\((\text{ft},s)\) is in the grammar, but \((\text{fi},s)\) is not.

\[\text{*kʃiŋts} \quad \text{(hypothetical)}\]
\[\text{ʃtijepus} \quad \text{‘he tells him’}\]
Relative Precedence

- \([ab]\) relatively precedes \([c]\) iff
  1. \([ab]\) precedes \([c]\) and
  2. no \([a]\) intervenes between \([ab]\) and \([c]\)

- The second conjunct captures the “nearest-preceding” aspect of the Chumash description above.

  \(\text{fi\textsuperscript{3}l\textsuperscript{3}usi\textsuperscript{3}si}\text{in} \ ‘\text{they (dual) are gone awry’}\)

- \([\text{fi}]\) precedes \([s]\)
- but \([\text{fi}]\) does not relatively precede \([s]\)

- Thus local blocking is achieved by not including \((\text{fi},s)\) in the grammar but including \((\text{ft},s)\).
Relative Precedence

- [ab] relatively precedes [c] iff
  1. [ab] precedes [c] and
  2. no [a] intervenes between [ab] and [c]

The second conjunct captures the “nearest-preceding” aspect of the Chumash description above.

\[ \text{[Si][lusisin ‘they (dual) are gone awry’} \]

- [ji] precedes [s]
- but [ji] does not relatively precede [s]

Thus local blocking is achieved by not including (ji,s) in the grammar but including (jt,s).
The learner simply records the relativized precedence bigram relations observed.

\[
\text{Precedence } G = \{ \}
\]

Sample = \{ \}

- The learner has already generalized: it accepts [\[i\], \[in\], \[lun\], \[lis\], \[sisisin\]]
- but not to words like [\[is\], \[ilus\]].
The learner simply records the relativized precedence bigram relations observed.

\[
\text{Precedence } G = \{(i\overline{j}, \overline{i}) \mid (i\overline{j}, 1), (i\overline{j}, u), (i\overline{j}, s), (i\overline{j}, i), (i\overline{j}, n), (\overline{i}l, u), (\overline{i}l, s), (\overline{i}l, i), (\overline{i}l, n), (\overline{l}u, s), (\overline{l}u, i), (\overline{l}u, n), (u\overline{s}, s), (u\overline{s}, i), (u\overline{s}, n), (s\overline{i}, s), (s\overline{i}, n), (i\overline{s}, i)\}
\]

Sample = \{ \overline{j}i\overline{f}\overline{u}\overline{s}\overline{i}\overline{s}\overline{i}\overline{n} \}

- The learner has already generalized: it accepts \[j\overline{i}\overline{f}, \overline{j}\overline{i}\overline{n}, \overline{j}\overline{l}\overline{u}\overline{n}, \overline{j}\overline{i}\overline{s}, \overline{s}\overline{i}\overline{s}\overline{i}\overline{n}\]\n- but not to words like \[j\overline{i}s, \overline{j}\overline{i}\overline{u}\overline{s}\].
The learner simply records the relativized precedence bigram relations observed.

\[
\text{Precedence } G = \begin{cases} 
(f_i,f) \\
(i,f,l) & (i,f,u) & (i,f,s) & (i,f,i) & (i,f,n) \\
(f,l,u) & (f,l,s) & (f,l,i) & (f,l,n) \\
(l,u,s) & (l,u,i) & (l,u,n) \\
(u,s,s) & (u,s,i) & (u,s,n) \\
(s,i,s) & \ (s,i,n) \\
(i,s,i) & \ 
\end{cases}
\]

Sample = \{ fiflusisin \}

- The learner has already generalized: it accepts [fif, fin, flun, flis, sisisin]
- but not to words like [fis, filus].
The learner simply records the relativized precedence bigram relations observed.

\[
\text{Precedence } G = \begin{cases} 
(i\bar{i},\bar{i}) \\
(i\bar{i},l) & (i\bar{i},u) & (i\bar{i},s) & (i\bar{i},i) & (i\bar{i},n) \\
(l\bar{l},u) & (l\bar{l},s) & (l\bar{l},i) & (l\bar{l},n) \\
(lu,s) & (lu,i) & (lu,n) \\
(us,s) & (us,i) & (us,n) \\
(si,s) & (si,n) \\
(is,i)
\end{cases}
\]

Sample = \{ if\bar{u}lusi\bar{s}in \}

- The learner has already generalized: it accepts [i\bar{f}, i\bar{n}, l\bar{l}un, l\bar{l}is, sis\bar{i}s\bar{i}] but not to words like [i\bar{s}, i\bar{l}u\bar{s}].
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