Control Synthesis for Discrete Event Systems
Final project - Ling 667

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1 Background - The Ramadge-Wonham Framework

2 Supervisory control - game theoretic approach

3 Results
Modeling of DES

A discrete event system (DES) is a dynamic system that evolves in accordance with the abrupt occurrence, at possible unknown irregular intervals, of physical events.

–[Ramadge1987]

A DES can be modeled as a deterministic finite state automaton (DFA):

\[
P = \langle Q, \Sigma = \Sigma_c \cup \Sigma_{uc}, T, q_0, F \rangle
\]

- $\Sigma_c$ the set of controllable actions/events;
- $\Sigma_{uc}$ is the set of uncontrollable actions/events.
A simple example of DES

\[ \Sigma = \{ \text{start (s), stop (t), breakdown (b), repair (r)} \} \]. \( q_0 = 1, F = \{1\} \).

Figure: An example of discrete event system where \( \Sigma_c = \{ \text{start, repair} \} \) and \( \Sigma_{uc} = \{ \text{stop, breakdown} \} \).
Supervisory Control for DES

Supervisor

- A supervisor $S$: $\text{Pre}(L(P)) \rightarrow 2^\Sigma$.
- $S$ cannot disable $\sigma \in \Sigma_{uc}$.

Specification

- A specification $\text{SPEC}$ is a DFA $C$ such that $L(C) \subseteq L(P)$.
- $L(C)$ is controllable iff $\exists S$ such that $P$ supervised by $S$, denoted $P/S$, yields $L(P/S) = L(C)$.
Let $\text{SPEC } C: L(C) = \{ \text{start breakdown repair} \}$.

Figure: A simple DES with a simple specification $C$. 
Problem and approach

Problem

*Determine the controllability of a given SPEC $C$, $L(C) \subseteq L(P)$ and if controllable, find $S$.***

Approach

- Partially synchronization product $P \circ C$.
- Game theoretic approach[Pin] to find $S$. 
Partially synchronization product

\[ P = \langle Q_1, T_1, \Sigma, q_01, F_1 \rangle \text{ and } C = \langle Q_2, T_2, \Sigma, q_02, F_2 \rangle. \]

Define \( P \circ C \) as:

- \( Q = Q_1 \times Q_2 \);
- \( T((q_1, q_2), \sigma) = \begin{cases} 
    (T_1(q_1, \sigma), q_2) & \text{if } T(q_1, \sigma) \downarrow \text{ but } T(q_2, \sigma) \uparrow, \\
    (T_1(q_1, \sigma), T(q_2, \sigma)) & \text{o.w.} 
\end{cases} \)
- \( q_0 = (q_{10}, q_{20}) \) and \( F \subseteq F_1 \times F_2 \).
Example-partially synchronization product

Figure: A simple DES with a simple specification $C$.

Figure: $P \circ C$. $s$ - start; $b$ - breakdown; $r$ - repair; $t$ - stop.
The supervisory control of DES can be transformed into a game between two players without restriction on turns.

Player 1 take a controllable action and player 2 choose to overrule this by taking an uncontrollable action.
For a given set $G \subseteq Q$, $q$ is in the attractor of $G$ if no uncontrollable action from $q$ leads to a state outside the attractor.

\[ \text{Attr}(14) = \{14, 33\}. \]
Algorithm for computing Attr($F$)

Starting with $W_0 = F$;

1. For each $q^* \in W_i$, find $Q = \{q \mid (\exists \sigma)[T(q, \sigma) = q^*]\}$;
2. Verify for each $q \in Q$, is there any $\sigma \in \Sigma_{uc}$ from $q$ such that $T(q, \sigma) \notin W_i$? If none, $W_{i+1} = W_i \cup \{q\}$.
3. The fixed-point $W_m$, i.e. $\forall k \geq m, W_k = W_m$ is the attractor of $F$.

Claim

A DES is controllable iff $q_0 \in \text{Attr}(F)$. 
Example - Another specification

\[ L(C_2) = \{ \text{start stop, start breakdown repair} \}. \]

Figure: A simple DES with a simple specification \( C \).

Computing \( \text{Attr}(15, 14) \):

1. \( W_0 = \{15, 14\} \);
2. \( W_1 = \{15, 14, 33\} \);
3. \( W_2 = \{15, 14, 33, 22\} \);
4. \( W_3 = \{15, 14, 33, 22, 11\} \);

Figure: \( P \circ C \). s - start; b - breakdown; r - repair; t - stop.
Work completed so far...

- syn_product: partially synchronization product.
- get_attr: attractor of $F$ for a given finite state automaton (FSA).
- is_ctrlable: verify whether there exists a supervisor for a given Spec.
- ctrller: for a given state, output a set of events that is enabled by the supervisor.

Further work: test the correctness for more complicated systems.
Dominique Perrin and Jean Érin Pin.  
*Infinite words: automata, semigroups, logic and games.*  

P J Ramadge and W M Wonham.  
Supervisory Control of a Class of Discrete Event Processes.  