Theorem 5.6 [The Master Theorem\textsuperscript{1}]: Let
\[
T(n) = \begin{cases} 
  c & \text{if } n < d, \\
  aT(n/b) + f(n) & \text{if } n \geq d,
\end{cases}
\]
where \(a \geq 1, b > 1,\) and \(d\) are integers and \(c\) is a positive constant. Let \(\nu = \log_b a.\)

**Case (i)** \(f(n)\) is “definitely smaller” than \(n^\nu\): If there is a small constant \(\epsilon > 0,\) such that \(f(n) \leq n^{\nu-\epsilon},\) that is, \(f(n) \ll n^\nu,\) then \(T(n) \sim n^\nu.\)

**Case (ii)** \(f(n)\) is “similar in size” to \(n^\nu\): If there is a constant \(k \geq 0,\) such that \(f(n) \sim n^{\nu}(\log n)^k,\) then \(T(n) \sim n^{\nu}(\log n)^{k+1}.\)

**Case (iii)** \(f(n)\) is “definitely larger” than \(n^\nu\): If there are small constants \(\epsilon > 0\) and \(\delta < 1,\) such that \(f(n) \geq n^{\nu+\epsilon}\) and \(af(n/b) \leq \delta f(n),\) for \(n \geq d,\) then \(T(n) \sim f(n).\)

Note the key roles that are played by the constants \(a, b,\) and \(\nu\) along with the function \(f(n)\) in the above theorem. The constants \(c\) and \(d\) are lesser importance in that they do not enter the conclusions except in case (iii) where \(d\) enters. Case (ii) is the one that we will use the most. The phrases “definitely smaller,” “similar in size,” and “definitely larger” are intended to give some intuitive idea of what the conditions on \(f(n)\) mean in each case. The detailed conditions are the ones that have to be applied however.

**Example.** Let
\[
T(n) = \begin{cases} 
  c & \text{if } n = 1, \\
  2T(n/2) + dn & \text{for some constant } d > 0.
\end{cases}
\]
Then \(\nu = \log_2 2 = 1\) and \(T(n) \sim n \log n\) by part (2), with \(k = 0,\) of the master theorem.

\textsuperscript{1}From page 268 of Goodrich and Tamassia.