A formal analysis of Correspondence Theory
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Introduction

This paper investigates the computational complexity of Correspondence Theory, as put forth by [1], which explicitly recognizes the correspondence between underlying and surface elements. Assuming Correspondence Theory, we have four distinct results:

- GEN is provably not MSO-definable.
- For a given input w, GEN(w) is provably MSO-definable.
- In fact, correspondence structures for a given input w are provably FO-definable with (<).
- Typical underlying representation (UR) to surface representation (SR) maps can be directly described with FO logic without recourse to optimization.

Our approach is similar in spirit to [2], which uses logic to formalize OT constraints. The major difference is that we employ language-specific inviolable constraints, cf. [3].

Correspondence Theory

The Correspondence Theory approach described in [1] characterizes GEN as a function which maps a UR to an input of ordered triples, as in (1).

\[ \text{Gen}(\text{abc}) = \{ (\text{abc}, [\text{abc}], \text{R}_1), (\text{abc}, [\text{abcd}], \text{R}_2), (\text{abc}, [\text{abc}], \text{R}_3), \ldots \} \]

Within each triple lies another relation (\text{R}_k), which maps the segments in the input to those in the output candidates. The fully faithful mapping is common (a_1,b_1,c_1 \rightarrow a_2,b_2,c_2), but there are unboundedly many other possible relations, some examples of which are shown in (2). Segments in the input can have zero, one, or multiple corresponding segments in an output candidate, and these segments may or may not be identical. The triples are sometimes represented as in (2).

\[ /a_1/b_1/c_1/ \rightarrow a_2/b_2/c_2 \]

The GEN function is not MSO-definable.

The core reason is stated in [4] as Fact 1.37, where \( U \) is an input candidate, and \( S \) is the set of output candidates:

1. For every monadic second-order transduction \( f \) there exists an index \( k \) such that, if \( f \) transforms a relational structure \( U \) into a relational structure \( S \), then \( |S| \leq k \times |U| \).

In other words, the size of the set of possible output candidates must be bounded in size. However, it is not.

The candidates are MSO-definable.

In fact, the set of output candidates created by Gen for a given input (e.g. /katəz/) can be modeled by using a set of FO-definable constraints (3)-(7) over a relational structure, with precedence relations.

\[ \text{edge}_c(x, y) \rightarrow [u(x) \land u(y)] \land \text{voiceless}(x) \land \text{voiceless}(y) \]

Typical mappings from UR to SR using FO constraints

In order to describe specific phonological alternations, we need language-specific, inviolable constraints on the relational structures described in our previous result.

Below we describe canonical processes of assimilation, insertion, and deletion, using this model. The constraints are formed as statements that ban the crucial substructures.

English Voice Assimilation

A voiced sound followed by a voiced obstruent:

\[ T_c(D(x)) = \{ (x, y) \mid \text{voiceless}(x) \land \text{voiceless}(y) \} \]

A voiceless sound followed by a voiceless obstruent:

\[ T_c(T(x)) = \{ (x, y) \mid \text{voiceless}(x) \land \text{voiceless}(y) \mid \text{voiceless}(y) \} \]

An underlyingly voiceless sound followed by a voiceless obstruent has to surface as one followed by a voiceless obstruent:

\[ (x, y) \mid T_c(T(x) \land u(x) \land \text{edge}_c(x, y)) \rightarrow T_c(T(y)) \]

Constraint (11) describes voice assimilation in English with a conditional statement. Alternatively, we can also describe the same process with banned substructures. Below are a few of them:

\[ \neg(3x,y) \mid T_c(T(x) \land u(x) \land \text{edge}_c(x, y) \land T_c(D(y)) \]

Tibetan Consonant-deletion

A voiceless sound followed by a voiced obstruent:

\[ T_c(D(x)) = \{ (x, y) \mid \text{voiceless}(x) \land \text{voiceless}(y) \} \]

An underlyingly voiceless sound followed by a voiceless obstruent has to surface as one followed by a voiceless obstruent:

\[ (x, y) \mid T_c(T(x) \land u(x) \land \text{edge}_c(x, y)) \rightarrow T_c(T(y)) \]

Constraint (12) describes the underlying voice assimilation in English with a conditional statement. Alternatively, we can also describe the same process with banned substructures. Below are a few of them:

\[ \neg(3x,y) \mid T_c(T(x) \land u(x) \land \text{edge}_c(x, y) \land T_c(D(y)) \]

References
